Multiple Choice Questions
Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. Suppose \( f(x) = \frac{9}{x+6} \). Find the value of \( A \), given that
\[
\frac{f(x+h) - f(x)}{h} = \frac{A}{(x+6)(x+h+6)}.
\]
Possibilities:
(a) \(-11\)
(b) \(-10\)
(c) \(-9\)
(d) \(-8\)
(e) \(-7\)

2. Suppose
\[
\frac{f(x+h) - f(x)}{h} = 3x + 3h + 7.
\]
Determine the slope of the tangent to the graph of \( y = f(x) \) at \( x = 3 \).
Possibilities:
\[
\text{Slope } T. L. = f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}
\]
(a) 12
(b) 13
(c) 14
(d) 15
(e) 16
3. Find $f'(x)$ where

\[ f(x) = \frac{1}{\sqrt[3]{x}} = x^{-\frac{1}{3}} \]

\[ f'(x) = \left( -\frac{1}{3} \right) x^{-\frac{4}{3}} - 1 \]

\[ = \left( -\frac{1}{3} \right) x^{-\frac{4}{3}} \]

Possibilities:
(a) $-\frac{1}{3}x^{-4/3}$
(b) $3x^2$
(c) $-3x^4$
(d) $-\frac{1}{3}x^{-2/3}$
(e) $\frac{1}{(1/3)x^{-2/3}}$

4. Suppose $F(x) = \frac{f(x)}{g(x)}$, $f(3) = 10$, $g(3) = 3$, $f'(3) = 3$, $g'(3) = 10$. Find $F'(3)$.

\[ F'(3) = \frac{f'(3)g(3) - f(3)g'(3)}{g(3)^2} \]

\[ = \frac{3 \cdot 3 - 10 \cdot 10}{3^2} = \frac{-91}{9} \]

Possibilities:
(a) $-91/9$
(b) $91/3$
(c) $91/9$
(d) $10/3$
(e) $-91/3$

5. Determine $H'(2)$, provided that $H(t) = (t^2 + 3t - 1)(t^2 - 3)$.

Possibilities:
(a) 1
(b) $-29$
(c) $43$
(d) 7
(e) 0

\[ H'(t) = \left( t^2 + 3t - 1 \right)' \left( t^2 - 3 \right) + \left( t^2 + 3t - 1 \right) \left( t^2 - 3 \right)' \]

\[ = (2t + 3)(t^2 - 3) + (t^2 + 3t - 1)(2t) \]

\[ H'(2) = (2 \cdot 2 + 3)(2^2 - 3) + (2^2 + 3 \cdot 2 - 1)(2 \cdot 2) \]

\[ = 7 \cdot 1 + 9 \cdot 4 = 43 \]
6. Find the derivative, \( f'(3) \), where

\[
f(x) = \sqrt{40 + x^2} = \left(40 + x^2\right)^{1/2}
\]

**Possibilities:**

(a) 1/7  
(b) 2/7  
(c) 3/7  
(d) 4/7  
(e) 5/7

So \( f'(3) = \frac{3}{\sqrt{49}} = \frac{3}{7} \).

7. Suppose \( F(G(x)) = x^3 \) and \( G'(2) = 3 \). Determine \( F'(G(2)) \).

**Possibilities:**

(a) 3  
(b) 12  
(c) 4  
(d) 1  
(e) 6

So \( 3x^2 = F'(G(x)) \cdot G'(x) \), hence \( F'(G(2)) = 4 \).

8. The tangent line to \( y = f(x) \) at \( x = 5 \) is given by

\[ y = -8(x - 5) + 11. \]

Determine the \( f(5) + f'(5) \). (Hint: use the tangent line to determine each of \( f(5) \) and \( f'(5) \). Then add.)

**Possibilities:**

(a) 3  
(b) -29  
(c) 8  
(d) -5  
(e) -11

\[
f'(5) = \text{Slope} = -8.
\]

\[
f'(5) \Rightarrow -8(5-5) + 11 = 11.
\]

So \( f(5) + f'(5) = -8 + 11 = 3 \).
9. Find the fourth derivative, \( f^{(4)}(x) \), where

\[
f(x) = 2x^5 - 9x^2
\]

**Possibilities:**

(a) \( 1250x^5 \)
(b) \( 28x - 18 \)
(c) \( 240x \)
(d) \( 1250x \)
(e) \( 28x \)

\[
f'(x) = 10x^4 - 18x
\]
\[
f''(x) = 40x^3 - 18
\]
\[
f'''(x) = 120x^2
\]
\[
f^{(4)}(x) = 240x
\]

10. Find the derivative, \( f'(t) \), where

\[
f(t) = e^{t^2+4t+7}
\]

**Possibilities:**

(a) \( (2t+4)e^{t^2+4t+7} \)
(b) \( e^{2t+4} \)
(c) \( \ln(t^2+4t+7) \)
(d) \( e^{t^2+4t+7} \)
(e) \( (t^2+4t+7)e^{t^2+4t+6} \)

\[
(2t+4)e^{t^2+4t+7}
\]

11. Find the derivative, \( f'(x) \), where

\[
f(x) = \ln(x^2+4x+3)
\]

**Possibilities:**

(a) \( x^2+4x+3 \)
(b) \( \frac{1}{x^2+4x+3} \)
(c) \( \frac{2x+4}{x^2+4x+3} \)
(d) \( 2x+4 \)
(e) \( \frac{x^2+4x+3}{2x+4} \)

\[
f'(x) = \frac{2x+4}{x^2+4x+3}
\]
12. Find the derivative, \( f'(38) \), where
\[
f(x) = x^2 + e^{-x}
\]
\[
f'(x) = 2x - e^{-x}
\]
\[
f'(38) = 2 \cdot 38 - e^{-38}
\]
\[
= 76 - e^{-38}
\]

**Possibilities:**
(a) \( 76 - 38e^{-37} \)
(b) \( 76 + e^{-38} \)
(c) \( 76 - e^{-38} \)
(d) \( 76 + 38e^{-38} \)
(e) \( 76 - 38e^{-39} \)

13. Find the 12th derivative, \( f^{(12)}(x) \), where
\[
f(x) = e^{3x}
\]
\[
f^{(12)}(x) = 3^{12} e^{3x}
\]

**Possibilities:**
(a) \( 0 \)
(b) \( 3^{12} e^{3x} \)
(c) \( 12 e^{3x} \)
(d) \( 12^3 e^{3x} \)
(e) \( 3e^{3x-1} \)

14. How much money must be invested now in order to have $5000 in 6 years, assuming interest is compounded continuously at an annual rate of 7.5%?

**Possibilities:**
(a) \( 5000 e^{0.450} \)
(b) \( 5000 e^{0.450} \)
(c) \( 5000 e^{-0.450} \)
(d) \( 5000 e^{-0.450} \)
(e) \( 5000 (1 + 0.08)^{-6} \)

\[
P(t) = P_0 e^{0.075t}
\]
\[
P(6) = 5000 = P_0 e^{0.075 \cdot 6}
\]
\[
5000 = P_0 e^{0.45}
\]
\[
5000 = \frac{5000}{e^{0.45}} = 5000 e^{-0.45}
\]
\[
= 5000 e^{-0.45}
\]
15. The population of a certain country doubles every 21 years. If we express the population as $P(t) = P_0 e^{rt}$, then find $r$.

**Possibilities:**

(a) $\frac{\ln (2)}{21}$

(b) $\frac{21}{\ln (2)}$

(c) $\frac{2}{\ln (21)}$

(d) $\frac{\ln (21)}{2}$

(e) $21 \cdot \ln (2)$

\[
P(21) = 2P_0 = P_0 e^{21r} \Rightarrow 2 = e^{21r} \Rightarrow \ln (2) = 21r \Rightarrow r = \frac{\ln (2)}{21}
\]

16. Find the maximum value of $f(x)$ on $[0, 4]$ where $f(x) = 2x^3 - 3x^2 - 12x$.

**Possibilities:**

(a) Maximum value = 32

(b) Maximum value = 4

(c) Maximum value = 0

(d) Maximum value = -20

(e) Maximum value = 7

\[
f'(x) = 0 \Rightarrow 6x^2 - 6x - 12 = 0 \Rightarrow x^2 - x - 2 = 0 \Rightarrow (x + 1)(x - 2) = 0 \Rightarrow x = -1 \text{ or } x = 2
\]

\[
f(0) = 2(0)^3 - 3(0)^2 - 12(0) = 0
\]

\[
f(2) = 2(2)^3 - 3(2)^2 - 12(2) = -20.
\]

\[
f(4) = 2(4)^3 - 3(4)^2 - 12(4) = 32.
\]

17. For which value of $x$ does $f(x) = 1 + 2x + 3x^2 + 4x^3 + 5x^4$ attain its maximum value, on the interval [2, 6]?

**Possibilities:**

(a) Maximum value attained at $x = 3$

(b) Maximum value attained at $x = 2$

(c) Maximum value attained at $x = 6$

(d) Maximum value attained at $x = 4$

(e) $f(x)$ does not attain a maximum value on [2, 6]

\[
f'(x) = 2 + 6x + 12x^2 + 20x^3
\]

For $x \in [2, 6]$, $f'(x)$ is always positive, so the maximum must be at an end point.

\[
f(2) = 129.
\]

\[
f(6) = 7965.
\]
18. According to the Extreme Value Theorem, which of the functions are guaranteed to attain a maximum value on the given interval?

- (I) A continuous function on \((-\infty, \infty)\)
- (II) A continuous function on \([-1, 3]\)
- (III) A continuous function on \([0, 9]\)

Possibilities:
- (a) Only (II) is guaranteed to have a maximum and a minimum.
- (b) (II) and (III) are guaranteed to have maxima and minima.
- (c) Only (I) is guaranteed to have a maximum and a minimum.
- (d) Only (III) is guaranteed to have a maximum and a minimum.
- (e) (I) and (III) are guaranteed to have maxima and minima.

19. Let \(f(x) = x^2 + 9\). Find a value \(c\) between \(x = 4\) and \(x = 8\) so that the average rate of change of \(f(x)\) from \(x = 4\) to \(x = 8\) is equal to the instantaneous rate of change of \(f(x)\) at \(x = c\).

\[
\frac{f(8) - f(4)}{8 - 4} = 12
\]

Possibilities:
- (a) 4
- (b) 5
- (c) 6
- (d) 7
- (e) 8

So \(2c = 12 \Rightarrow c = 6\)

20. Suppose \(g(x) = 2x^3\) and the tangent line to \(y = f(x)\) at \(x = 2\) is given by \(y = 5(x - 2) + 5\). Determine the slope of the tangent line to \(y = f(x) \cdot g(x)\) at \(x = 2\).

Possibilities:
- (a) 200
- (b) 80
- (c) 29
- (d) 21
- (e) 120

\[
y' = f'(2)g(2) + f(2)g'(2)
\]
\[
f'(2) = 5 \text{ (slope of tangent line)}
\]
\[
g(2) = 2 \cdot 2^3 = 16
\]
\[
g'(2) = 2 \cdot 3 \cdot 2 = 12
\]

So \(y' = 5 \cdot 16 + 5 \cdot 24 = 200\).