Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (b) is correct, you must write:

a □ c □ d □ e □

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1. □ □ □ □ □
2. □ □ □ □ □
3. □ □ □ □ □
4. □ □ □ □ □
5. □ □ □ □ □
6. □ □ □ □ □
7. □ □ □ □ □
8. □ □ □ □ □
9. □ □ □ □ □
10. □ □ □ □ □

11. □ □ □ □ □
12. □ □ □ □ □
13. □ □ □ □ □
14. □ □ □ □ □
15. □ □ □ □ □
16. □ □ □ □ □
17. □ □ □ □ □
18. □ □ □ □ □
19. □ □ □ □ □
20. □ □ □ □ □

For grading use:

<table>
<thead>
<tr>
<th>Number Correct</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>(out of 20 problems)</td>
<td>(out of 100 points)</td>
</tr>
</tbody>
</table>
Please make sure to list the correct section number on the front page of your exam. In case you forgot your section number, consult the following table. Your section number is determined by your recitation time and location.

<table>
<thead>
<tr>
<th>Section #</th>
<th>Instructor</th>
<th>Recitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>001</td>
<td>D. Akers</td>
<td>T 8:00 am - 9:15 am, CB 243</td>
</tr>
<tr>
<td>002</td>
<td>D. Akers</td>
<td>R 8:00 am - 9:15 am, CB 243</td>
</tr>
<tr>
<td>003</td>
<td>D. Akers</td>
<td>T 12:30 pm - 1:45 pm, TEB 231</td>
</tr>
<tr>
<td>004</td>
<td>Q. Liang</td>
<td>R 9:30 am - 10:45 am, NURS 502A</td>
</tr>
<tr>
<td>005</td>
<td>Q. Liang</td>
<td>T 11:00 am - 12:15 pm, CB 243</td>
</tr>
<tr>
<td>006</td>
<td>Q. Liang</td>
<td>R 11:00 am - 12:15 pm, CB 243</td>
</tr>
<tr>
<td>007</td>
<td>D. Corral</td>
<td>T 2:00 pm - 3:15 pm, DH 301</td>
</tr>
<tr>
<td>008</td>
<td>D. Corral</td>
<td>R 2:00 pm - 3:15 pm, DH 301</td>
</tr>
<tr>
<td>009</td>
<td>D. Corral</td>
<td>T 11:00 am - 12:15 pm, DH 353</td>
</tr>
<tr>
<td>010</td>
<td>A. Barra</td>
<td>R 11:00 am - 12:15 pm, DH 353</td>
</tr>
<tr>
<td>011</td>
<td>A. Barra</td>
<td>T 12:30 pm - 1:45 pm, MMRB 243</td>
</tr>
<tr>
<td>012</td>
<td>A. Barra</td>
<td>R 12:30 pm - 1:45 pm, MMRB 243</td>
</tr>
<tr>
<td>013</td>
<td>J. Jung</td>
<td>T 11:00 am - 12:15 pm, TPC 113</td>
</tr>
<tr>
<td>014</td>
<td>J. Jung</td>
<td>R 11:00 am - 12:15 pm, TPC 113</td>
</tr>
<tr>
<td>015</td>
<td>F. Camacho</td>
<td>T 12:30 pm - 1:45 pm, CB 219</td>
</tr>
<tr>
<td>016</td>
<td>J. Jung</td>
<td>R 12:30 pm - 1:45 pm, CB 219</td>
</tr>
<tr>
<td>017</td>
<td>F. Camacho</td>
<td>T 2:00 pm - 3:15 pm, FB 88</td>
</tr>
<tr>
<td>018</td>
<td>F. Camacho</td>
<td>R 2:00 pm - 3:15 pm, TPC 212</td>
</tr>
<tr>
<td>019</td>
<td>S. Hamilton</td>
<td>T 3:30 pm - 4:45 pm, CP 345</td>
</tr>
<tr>
<td>020</td>
<td>S. Hamilton</td>
<td>R 3:30 pm - 4:45 pm, BE 301</td>
</tr>
<tr>
<td>021</td>
<td>S. Hamilton</td>
<td>T 2:00 pm - 3:15 pm, CB 340</td>
</tr>
<tr>
<td>022</td>
<td>J. Constable</td>
<td>R 2:00 pm - 3:15 pm, CB 345</td>
</tr>
<tr>
<td>023</td>
<td>J. Constable</td>
<td>T 9:30 am - 10:45 am, L 201</td>
</tr>
<tr>
<td>024</td>
<td>J. Constable</td>
<td>R 9:30 am - 10:45 am, L 201</td>
</tr>
<tr>
<td>025</td>
<td>M. Shaw</td>
<td>MWF 9:00 am - 9:50 am, CB 110</td>
</tr>
</tbody>
</table>
Multiple Choice Questions

Show all your work on the page where the question appears. Clearly mark your answer both on the cover page on this exam and in the corresponding questions that follow.

1. Suppose \( f(x) = x^4 - 8x^3 - 30x^2 + 2x - 3 \). Find the largest interval or collection of intervals on which \( f(x) \) is concave down.

   \[ f''(x) \leq 0. \]

   Possibilities:
   - (a) \((-\infty, -1)\) and \((5, \infty)\)
   - (b) \((-1, 5)\)
   - (c) \((-\infty, -1)\)
   - (d) \((-1, \infty)\)
   - (e) \((5, \infty)\)

   \[ f'(x) = 4x^3 - 24x^2 - 60x + 2 \]
   \[ f''(x) = 12x^2 - 48x - 60 \]
   \[ f''(x) = 12(x - 5)(x + 1) \]
   \[ f''(x) = 0 \Rightarrow x = 5 \quad \text{or} \quad x = -1. \]
   - Concave down \([-1, 5]\)

2. Suppose the derivative of \( g(t) \) is \( g'(t) = (t + 9)(t - 3)(t - 1) \). Determine the largest interval or collection of intervals on which \( g(t) \) is decreasing.

   \[ g'(t) < 0. \]

   Possibilities:
   - (a) \((-\infty, -9)\) and \((1, 3)\)
   - (b) \((3, \infty)\)
   - (c) \((-9, 1)\) and \((3, \infty)\)
   - (d) \((-\infty, -9)\) and \((3, \infty)\)
   - (e) \((-9, 3)\)

3. Determine the interval or collection of intervals on which \( y = f(x) \) is decreasing. Please note the graph is of the derivative of \( y = f(x) \).

   \[ f'(x) < 0. \]

   \[ f'(x) < 0 \quad \text{below x-axis} \]

   Possibilities:
   - (a) \( f(x) \) is never decreasing
   - (b) \((-\infty, 0), (3, 4)\), and \((6, \infty)\)
   - (c) \((-\infty, 1)\) and \((3.5, 5)\)
   - (d) \((0, 3)\) and \((4, 6)\)
   - (e) \((1, 3.5)\) and \((5, \infty)\)
4. Suppose that \( y = f(x) \) is continuous and differentiable for all real numbers, and \( f'(x) < 0 \) for all \( x \). Which of the following must be true about \( y = f(x) \)?

**Possibilities:**

(a) \( y = f(x) \) is always decreasing.
(b) \( y = f(x) \) is always concave down.
(c) \( y = f(x) \) is always below the \( x \)-axis.
(d) \( y = f(x) \) is always increasing.
(e) \( y = f(x) \) is always concave up.

5. Suppose that \( f(x) = x^3 - 9x^2 + 48x + 9 \). Determine the \( x \) coordinate of the inflection point of \( y = f(x) \).

\[
\begin{align*}
f'(x) &= 3x^2 - 18x + 48, \\
f''(x) &= 6x - 18 = 6(x - 3) \\
\text{Concavity changes.}
\end{align*}
\]

**Possibilities:**

(a) \( x = 2 \)
(b) \( x = 3 \)
(c) \( x = 4 \)  \[
\begin{array}{cccc}
\text{f''(x)} & \text{---} & \text{+++} & \text{---} \\
\text{C.D.} & \text{C.U.}
\end{array}
\]
(d) \( x = 5 \)
(e) \( x = 6 \)

6. The product of two positive numbers, \( x \) and \( y \) is 19. Determine the maximum value of the expression \( x + 2y \).

**Possibilities:**

(a) \( \sqrt{38} \)
(b) \( 19 \)
(c) \( 2\sqrt{38} \)
(d) \( 3\sqrt{38} \)
(e) \( 38 \)

As stated, this problem has **no solution**. It should have read "Determine the minimum value... legitimate attempts to find either the max. OR min were granted credit."
Revised #6.
The product of two positive numbers, \( x \) and \( y \), is 19. Determine the minimum value of the expression \( x + 2y \).

Solution.
\[ xy = 19 \Rightarrow y = \frac{19}{x} = 19x^{-1} \]

We want \( x + 2y = x + 2 \cdot 19x^{-1} = x + 38x^{-1} \) minimal.

But \( (x + 38x^{-1})' = 1 - 38x^{-2} = 0 \)
\[ \Rightarrow 1 = 38x^{-2} \]
\[ \Rightarrow x^2 = 38 \]
\[ \Rightarrow x = \pm \sqrt{38} \]
(But \( x \) should be positive)

So \( x = \sqrt{38} \)

So \( x + 2y = \sqrt{38} + \frac{38}{\sqrt{38}} = 2\sqrt{38} \).
7. Determine the area of the largest rectangle which has one corner at the origin (0,0) and opposite corner in the first quadrant on the line \( y = -5x + 30 \).

**Possibilities:**

(a) 90
(b) 45
(c) 450
(d) 180
(e) 3/2

So \( A(3) = (-5 \cdot 3 + 30) \cdot 3 = 45 \)

8. Find the \( x \)-coordinate of the point on the curve \( y = \sqrt{x} \) which is closest to the point (14,0)

**Possibilities:**

(a) \( \sqrt{7} \)
(b) 14
(c) \( \sqrt{14} \)
(d) 28
(e) 27/2

Now, \( D = \sqrt{(x-14)^2 + (\sqrt{x} - 0)^2} \)

Now, \( A' = \frac{2(x-14) + 1}{2 \sqrt{(x-14)^2 + x}} = 0 \) \( \Rightarrow 2(x-14) + 1 = 0 \) \( \Rightarrow x = \frac{27}{2} \).

9. Determine the rate of increase of the area of a circle when the radius of the circle is 14 feet and the radius is increasing at the rate of 7 feet per minute.

**Possibilities:**

(a) 140\( \pi \) square feet per minute
(b) 98\( \pi \) square feet per minute
(c) 196\( \pi \) square feet per minute
(d) 49\( \pi \) square feet per minute
(e) 28\( \pi \) square feet per minute

\[ \frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt} \]

\[ = \pi \cdot 2 \cdot 14 \cdot 7 = 196 \pi \]
10. The price of a share of stock is increasing at a rate of 19 dollars per share per year. An investor is buying stock at a rate of 11 shares per year. How fast is the value of the investor’s stock growing when the price of the stock is 63 dollars per share and the investor owns 50 shares of the stock? (Hint: Write down an expression for the total value, \( V \), of the stock owned by the investor.)

Possibilities:
(a) $950 per year.
(b) $1643 per year.
(c) $1747 per year.
(d) $3150 per year.
(e) $209 per year.

\[
\frac{dV}{dt} = \frac{d}{dt}(pn) = p \frac{dn}{dt} + n \frac{dp}{dt}
\]

\[
= 19 \times 50 + 63 \times 11 = \$1643.
\]

11. An expanding rectangle has its length always equal to twice its width. The area is increasing at a rate of 72 square feet per minute. At what rate is the width increasing when the width is 9 feet?

Possibilities:
(a) 36 feet per minute.
(b) 18 feet per minute.
(c) 2 feet per minute.
(d) 8 feet per minute.
(e) 162 feet per minute.

\[
A = lw = (2w)w = 2w^2
\]

\[
\frac{dA}{dt} = 2w \frac{dw}{dt}
\]

\[
\text{Put } \frac{dA}{dt} = 72
\]

\[
\Rightarrow \frac{dw}{dt} = \frac{72}{2 \times 9} = 2
\]

12. The derivative of \( f(x) \) is

\[
f'(x) = (x + 3)(x + 2)(x - 4)(x - 5).
\]

Which of the following are true?

- (I) \( f(x) \) has a local maximum at \( x = -2 \)
- (II) \( f(x) \) has a local maximum at \( x = 4 \)
- (III) \( f(x) \) has a local minimum at \( x = 5 \)

Possibilities:
(a) (II) and (III) true
(b) Only (III) is true.
(c) Only (I) is true.
(d) Only (II) is true.
(e) (I) and (II) true.
13. Estimate the area under the graph of \( f(x) = x^2 + 5x \) for \( x \) between 0 and 2. Use a partition that consists of 4 equal subintervals of \([0, 2] \) and use the left endpoint of each subinterval as the sample point.

**Possibilities:**

(a) 6
(b) \( \frac{37}{4} \)
(c) \( \frac{65}{4} \)
(d) \( \frac{37}{2} \)
(e) 20

\[ f(0) = 0 \]
\[ f(0.5) = (0.5)^2 + 5(0.5) = 2.75 \]
\[ f(1) = 1^2 + 5(1) = 6 \]
\[ f(1.5) = 1.5^2 + 5(1.5) = 9.75 \]

Area = \( \frac{1}{2} (0 + 2.75 + 6 + 9.75) = 9.25 = \frac{37}{4} \)

14. Suppose you are given the data points for a function \( g(t) \):

\[
\begin{array}{c|c|c|c}
 t & 0 & 1 & 2 \\
g(t) & 9 & 12 & 16 \\
\end{array}
\]

If \( g(t) \) is a linear function on each interval between the given points, find

\[ \int_0^2 g(t) \, dt \]

**Possibilities:**

(a) \( \frac{25}{2} \)
(b) 21
(c) 37
(d) \( \frac{49}{2} \)
(e) \( \frac{49}{2} \)

\[ \text{Widths} = 1 \]

\[ \text{Area} = \left[ \frac{9 + 12}{2} + \frac{12 + 16}{2} \right] \]

(Not drawn to scale)
15. Evaluate the sum

\[ \sum_{k=1}^{8} k^3 = 6^3 + 7^3 + 8^3 = 1071 \]

**Possibilities:**
(a) 1068
(b) 1069
(c) 1070
(d) 1071
(e) 1072

Only 3 terms, simplest to just expand the sum.

16. Evaluate the sum

\[ \sum_{k=1}^{45} (k^2 + k) = \sum_{k=1}^{45} k^2 + \sum_{k=1}^{45} k \]

Use sum formulas!

\[ = \frac{45(45+1)(2\cdot45+1)}{6} + \frac{45 \cdot (45+1)}{2} \]

\[ = 32430 \]

17. Evaluate the sum

\[ 15 + 20 + 25 + 30 + \ldots + 250 + 255 \]

\[ = 5 \left( \frac{3+4+5+\ldots+50+51}{2} \right) \]

\[ = 5 \left[ \frac{1+2+3+4+\ldots+40+51}{2} - 1 - 2 \right] \]

\[ = 5 \left[ \frac{41 \cdot 50}{2} - 3 \right] \]
18. Suppose that the integral \( \int_{32}^{42} f(x) \, dx \) is estimated by the sum \( \sum_{k=1}^{N} f(32 + k \Delta x) \cdot \Delta x \). The terms in the sum equal areas of rectangles obtained using right endpoints of the subintervals of length \( \Delta x \) as sample points. If \( N = 500 \) equal subintervals are used, what is the value of \( \Delta x \)?

**Possibilities:**
(a) \( \Delta x = 0.02 \)
(b) \( \Delta x = 0.03 \)
(c) \( \Delta x = 0.04 \)
(d) \( \Delta x = 0.05 \)
(e) \( \Delta x = 0.06 \)

\[
\Delta x = \frac{42 - 32}{500} = \frac{10}{500} = 0.02
\]

19. Suppose that the integral \( \int_{9}^{30} x^2 \, dx \) is estimated by the sum \( \sum_{k=1}^{N} (9 + k \Delta x)^2 \cdot \Delta x \). The terms in the sum equal areas of rectangles obtained using right endpoints of the subintervals of length \( \Delta x \) as sample points. If \( N = 42 \) equal subintervals are used, what is area of the second rectangle?

**Possibilities:**
(a) 100
(b) 81/2
(c) 50
(d) 361/8
(e) 361/4

\[
\Delta x = \frac{30 - 9}{42} = \frac{21}{42} = \frac{1}{2}
\]

20. Estimate the area under the graph of \( y = \frac{1}{x} \) for \( x \) between 1 and 50 by dividing the interval \([1, 50]\) into 49 equal subintervals and using the left endpoint of each subinterval as sample point. Next, estimate the area using the right endpoint as sample point. Find the difference between the two estimates (left endpoint estimate minus right endpoint estimate).

**Possibilities:**
(a) 47/48
(b) 48/49
(c) 49/50
(d) 50/51
(e) 51/52

\[
= 1 - \frac{1}{50} = \frac{49}{50}
\]
Some Formulas

1. Summation formulas:
   \[ \sum_{k=1}^{n} k = \frac{n(n + 1)}{2} \]
   \[ \sum_{k=1}^{n} k^2 = \frac{n(n + 1)(2n + 1)}{6} \]

2. Areas:
   (a) Triangle \( A = \frac{bh}{2} \)
   (b) Circle \( A = \pi r^2 \)
   (c) Rectangle \( A = lw \)
   (d) Trapezoid \( A = \frac{b_1 + b_2}{2} h \)

3. Volumes:
   (a) Rectangular Solid \( V = lwh \)
   (b) Sphere \( V = \frac{4}{3} \pi r^3 \)
   (c) Cylinder \( V = \pi r^2 h \)
   (d) Cone \( V = \frac{1}{3} \pi r^2 h \)

4. Distance:
   (a) Distance between \((x_1, y_1)\) and \((x_2, y_2)\)
   \[ D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]