MA123 Final Exam

12 December 2006

NAME ____________________________ Section __________

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<thead>
<tr>
<th>Problem</th>
<th>Answer</th>
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Instructions. Circle your answer in ink on the page containing the problem and on the cover sheet. After the exam begins, you may not ask a question about the exam. Be sure you have all pages (containing 15 problems) before you begin. You will find a table of logarithms at the end of the exam that you need for Problem 6.
1. Find the average rate of change of the function $G(t) = |t^2 - 1|$ as $t$ changes from $-1$ to $2$.
   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) 4

2. Find the equation of a line tangent to the curve $y = 2x^2 + x + 1$ at $x = 2$.
   (a) $y = 9 + 11(x - 2)$
   (b) $y = 11 + 9(x - 2)$
   (c) $y = 22 + 13(x - 3)$
   (d) $y = 13 + 22(x - 3)$
   (e) $y = 7 + (4x + 1)$

3. Which horizontal line is tangent to the graph of $y = x^3 - x^2 - x + 2$?
   (a) $y = 0$
   (b) $y = 1$
   (c) $y = 2$
   (d) $y = 3$
   (e) $y = 4$
4. What is the largest value of $A$ such that the function $f(t)$ is defined on the interval $(0, A)$ where

$$f(t) = \frac{1}{t^3 - 8}$$

(a) 1  
(b) 2  
(c) 3  
(d) 4  
(e) 5

5. Find the limit as $x$ tends to 0 from the left

$$\lim_{x \to 0^-} \frac{|x|}{2x}$$

(a) $1/3$  
(b) $1/2$  
(c) 0  
(d) $-1/2$  
(e) $-1/3$

6. Suppose the position $P(t)$ of an object at time $t$ is given by $t^2 + 1$. Find a value of $t$ at which the instantaneous speed of the object equals the average speed on the interval $[0, 1]$.

(a) $1/3$  
(b) $1/2$  
(c) $2/3$  
(d) $3/4$  
(e) 1
7. Let \([x]\) denote the greatest integer function. Recall the definition:

\([x]\) equals the greatest integer less than or equal to \(x\).

How many points are there in the interval \((\frac{1}{2}, \frac{9}{2})\) where the derivative of \([x]\) is not defined?

(a) 1  
(b) 2  
(c) 3  
(d) 4  
(e) 5

8. Let

\[ f(x) = \begin{cases} 
-1 & \text{if } x < 0 \\
2 & \text{if } 0 \leq x < 2 \\
3 & \text{if } 2 \leq x 
\end{cases} \]

Evaluate the integral

\[ \int_{-2}^{3} f(x) \, dx \]

(a) 1  
(b) 2  
(c) 3  
(d) 4  
(e) 5

9. Evaluate the derivative, \(H'(1)\) if

\[ H(s) = \frac{2s}{s+1} \]

(a) 2/9  
(b) 4/9  
(c) 1/2  
(d) 3/2  
(e) 8/9
10. Use the summation formula below to evaluate \(3^3 + 4^3 + \cdots + 40^3\)

\[
\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}
\]

(a) 608391  
(b) 608400  
(c) 653479  
(d) 672400  
(e) 672391

11. Evaluate the limit

\[
\lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{n} f\left(\frac{k}{n}\right)
\]

where \(f(x) = x^2\). Hint: Draw a picture and relate the limit to an integral.

(a) \(1/5\)  
(b) \(1/4\)  
(c) \(1/3\)  
(d) \(1/2\)  
(e) 1

12. Evaluate the limit

\[
\lim_{h \to 0} \frac{f(3 + h) - f(3)}{h}
\]

where

\[f(x) = \sqrt{x + 1}\]

(a) \(1/6\)  
(b) \(1/5\)  
(c) \(1/4\)  
(d) \(1/3\)  
(e) \(1/2\)
13. Find the area of the rectangle of minimum area with one vertex (corner) at \((0, 0)\) and opposite corner on the ellipse \(x^2 + 4y^2 = 4\).

(a) \(3/4\)
(b) \(\sqrt{5}/4\)
(c) \(\sqrt{7}/4\)
(d) \(9/4\)
(e) \(\sqrt{11}/4\)

14. What is the average of the function \(h(t) = t^3 + 1\) on the interval \([1, 4]\) ?

(a) \(\frac{247}{12}\)
(b) \(\frac{257}{12}\)
(c) \(\frac{267}{12}\)
(d) \(\frac{277}{12}\)
(e) \(\frac{279}{12}\)

15. Suppose the cost, \(C(q)\), of stocking a quantity \(q\) of a product equals

\[
C(q) = \frac{100}{q} + q
\]

The rate of change of the cost with respect to \(q\) is called the marginal cost. When is the marginal cost positive?

(a) \(q > 10\)
(b) \(q > 15\)
(c) \(q < 20\)
(d) \(q < 25\)
(e) \(q = 30\)
Geometric Formulas

1. Areas
   
   (a) Triangle $A = \frac{bh}{2}$
   
   (b) Circle $A = \pi r^2$
   
   (c) Rectangle $A = lw$
   
   (d) Trapezoid $A = \frac{b_1+b_2}{2}h$

2. Volumes
   
   (a) Rectangular Solid $V = lwh$
   
   (b) Sphere $V = \frac{4}{3}\pi r^3$
   
   (c) Cylinder $V = Bh$
   
   (d) Cone $V = \frac{1}{3}Bh$