MA123 Exam 1

February 06 2008

NAME ____________________________ Section __________

Problem  Answer
1    a b c d e
2    a b c d e
3    a b c d e
4    a b c d e
5    a b c d e
6    a b c d e
7    a b c d e
8    a b c d e
9    a b c d e
10   a b c d e
11   a b c d e
12   a b c d e
13   a b c d e
14   a b c d e
15   a b c d e

Instructions. Circle your answer in ink on the page containing the problem and on the cover sheet. After the exam begins, you may not ask a question about the exam. Be sure you have all pages (containing 15 problems) before you begin. A list of formulas that may be useful for this exam is on the last page (you may tear this page off if you wish).

For grading use:

Number of problems correct: ___________/15

SCORE: ___________/100
1. Find the domain of the function 

\[ F(s) = \frac{1}{\sqrt{s^2 - 1}} \]

(a) All \( s \) such that either \( -\infty < s < -1 \) or \( 1 < s < \infty \)
(b) All \( s \) such that \( -1 < s < 1 \)
(c) All \( s \) such that \( -\infty < s < \infty \)
(d) All \( s \) such that either \( -\infty < s < 1 \) or \( 1 < s < \infty \)
(e) All \( s \) such that \( 0 < s < 1 \)

2. If the line given by \( s = A + B(t - 1) \) is perpendicular to the line \( s = t \) and contains the point \((1, 6)\) in the \((t, s)\)-plane, then

(a) \( A = 1, B = 4 \)
(b) \( A = 4, B = 1 \)
(c) \( A = 1, B = 6 \)
(d) \( A = 4, B = -1 \)
(e) \( A = 6, B = -1 \)

3. Find the average rate of change of the function

\[ R(t) = \sqrt{2t + 7} \]

as \( t \) changes from 1 to 9.

(a) \( \frac{1}{3} \)
(b) \( \frac{1}{2} \)
(c) \( \frac{1}{4} \)
(d) 4
(e) 3
4. Which of the following is true for the function \( f(x) = |x - 1| \)?

(a) \( f \) is differentiable at \( x = 1 \) and \( x = 2 \).  
(b) \( f \) is differentiable at \( x = 1 \), but not at \( x = 2 \).  
(c) \( f \) is differentiable at \( x = 2 \), but not at \( x = 1 \).  
(d) \( f \) is not differentiable at either \( x = 1 \) or \( x = 2 \).  
(e) None of the above.

5. Suppose the height of an object above ground at time \( t \) (in seconds) is measured by \( h(t) \) (in feet). If  
\[
h(t) = -16t^2 + 40t + 120,
\]
what is the speed of the object at time \( t = 0 \)?

(a) 16 feet per second  
(b) 32 feet per second  
(c) 120 feet per second  
(d) 40 feet per second  
(e) 56 feet per second

6. Find  
\[
\lim_{r \to 1} \frac{r^2 - 3r + 2}{r - 1}
\]

(a) 1  
(b) 0  
(c) −1  
(d) 2  
(e) Does not exist
7. Consider a triangle with base \( x \) and height \( 2x \). Find the instantaneous rate of change of the area of the triangle with respect to \( x \) when \( x = 5 \).

(a) 1 
(b) 2 
(c) 5 
(d) 10 
(e) 20

8. For the function 
\[
f(x) = \begin{cases} 
4x^2 - 1 & \text{if } x < 1 \\
3x + 2 & \text{if } x \geq 1 
\end{cases}
\]
find 
\[
\lim_{x \to 1^+} f(x)
\]

(a) 5 
(b) 3 
(c) 1 
(d) 0 
(e) Does not exist

9. Let \( g(s) = s^2 - 3s + 1 \). Find a value \( A \geq 0 \) such that the average rate of change of \( g(s) \) from 0 to \( A \) equals 8.

(a) 0 
(b) 8 
(c) 11 
(d) 15 
(e) 22
10. Find all values of $a$ such that the function

$$f(x) = \begin{cases} 
    x^2 + 2x & \text{if } x < a \\
    -1 & \text{if } x \geq a
\end{cases}$$

is continuous everywhere.

(a) $a = -1$ only
(b) $a = -2$ only
(c) $a = -1$ and $a = 1$
(d) $a = -2$ and $a = 2$
(e) all real numbers

11. Let $f(t) = 3t^2 + 6t + 1$. Find the value of $t$ for which the tangent line to the graph of $f(t)$ has slope 1.

(a) $-1$
(b) $-\frac{5}{6}$
(c) 0
(d) $\frac{7}{6}$
(e) 6

12. A train travels from city $A$ to city $B$ to city $C$. The distance from $A$ to $B$ is 20 miles. The distance from $B$ to $C$ is 45 miles. The train took 1 hour for the trip from $A$ to $B$, stopped at city $B$ for 30 minutes, and then went from $B$ to $C$ at an average speed of 30 miles per hour. What was the average speed of the train for the entire trip (in miles per hour)?

(a) 65
(b) 25
(c) $\frac{65}{7}$
(d) 50
(e) $\frac{65}{3}$
13. Find the instantaneous rate of change of the function \( H(t) = t^3 \) at \( t = 2 \).

(a) 2
(b) 3
(c) 8
(d) 12
(e) 27

14. If \( R(t) = t + 2 \) and \( R(Q(t)) = t \) then

(a) \( Q(t) = 2t \)
(b) \( Q(t) = t \)
(c) \( Q(t) = t - 2 \)
(d) \( Q(t) = t + 2 \)
(e) \( Q(t) = 2 - t \)

15. Suppose the cost \( C(q) \) (in dollars) of producing a quantity \( q \) of a product equals

\[
C(q) = 500 + 2q + \frac{1}{5}q^2
\]

The marginal cost \( M(q) \) equals the instantaneous rate of change of the total cost. Find the marginal cost when a quantity of 10 items are being produced.

(a) 2
(b) 6
(c) 10
(d) 20
(e) 500
List of Formulas:

1. You may use the following formula for the derivative of a quadratic function. If
\[ p(x) = Ax^2 + Bx + C \]
then
\[ p'(x) = 2Ax + B \]

2. The area of a triangle with base \( b \) and height \( h \) is \( \frac{1}{2}bh \).

3. If you cover a distance of \( d \) miles in \( t \) hours at a rate of \( r \) miles per hour, then \( d = rt \).