Friday, Feb 5, 2016

1. See Announcements on Canvas about EXAM.

2. SIGN ATTENDANCE on Canvas SHEET.

3. With your neighbors:
   - Discuss your "concept maps" for the topic of sequences and series. What are the key examples?
   - What are the most and least challenging topics for you?
Continue discussion about R. of C.

Remark: For \( F(x) = \sum_{n=0}^{\infty} a_n (x-c)^n \) with radius of convergence \( R \), i.e. (for all \( x \) satisfying \( |x-c| < R \), \( F(x) \) converges), \( F(x) \) has an interval of convergence that is one of the following:

\( (c-R, c+R) \) \( [c-R, c+R) \)
\( (c-R, c+R] \) \( [c-R, c+R] \)

So, we define the Int. of Conv. to be this choice above....
Q: c is always in Int. of Conv. Why?

A: \( F(c) = \sum_{n=0}^{\infty} a_n (c-c)^n \)

\( \frac{a_0 (c-c)^0 + a_1 (c-c)^1 + a_2 (c-c)^2 + \cdots}{a_0 (c-c)^0 + a_0 + a_2 + \cdots} \)

All are equal to 0.

Aside: Why is \( a_0 = 1 \) for any \( a_0 \)?
Two reasons why $a^0 = 1$.

1. (Intuitive)
   - When dividing a to the power of 0, it equals 1.
   - $a^0 = a^{-1} \cdot a^1$, but $a^{-1} \cdot a^1 = 1$.

2. (Formal)
   - We define 1 to be the multiplicative identity.
   - So, by $a^0 = 1$, it holds true for all $a$. 

So, we define 1 to be this identity.
Use power series
\[ e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \]

For use in equation (1.4), you get 4.

For \( x = 0 \), all terms where \( n > 0 \) are zero.

\[ x = \ln(a) \]

\[ 1 + \ln(a)^2 + \frac{\ln(a)^3}{3!} + \ldots \]

\[ a = e^{\ln(a)} = \ln(a) \]

\[ 1 + \ln(a)^2 + \frac{\ln(a)^3}{3!} + \ldots \]
Ex: Use ratio test to find int. of cmv.

For

\[ \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{\sqrt{n^2+1}} \]

Ratio test: \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \frac{(-1)^{n+1} x^{n+1}}{\sqrt{(n+1)^2+1}} \cdot \frac{\sqrt{n^2+1}}{(-1)^n x^n} \)

\[ = \lim_{n \to \infty} \frac{|x| \sqrt{n^2+1}}{\sqrt{(n+1)^2+1}} = |x| \cdot \lim_{n \to \infty} \frac{\sqrt{n^2+1}}{\sqrt{n^2+2n+2}} = |x| \cdot 1 = |x| \]

\( |x| < 1 \). \hspace{1cm} \text{This will yield abs. convergence by ratio test.}
This gives us |x| < 1, i.e. x ∈ (-1, 1).

But, we need to check endpoints, x = 1, -1.

For x = -1, we get
\[ \sum_{n=0}^{\infty} \frac{(-1)^n (-1)^n}{\sqrt{n^2+1}} = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}}. \]

Exercise: * diverges by comparison test with \( \sum_{n=1}^{\infty} \frac{1}{n} \). So, x = -1 is not in Int. of Conv.

For x = 1, we get
\[ \sum_{n=0}^{\infty} \frac{(-1)^n (1)^n}{\sqrt{n^2+1}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n^2+1}}. \]

Exercise: * converges by alt. series test.

So, Int. of Conv. is (-1, 1].
Stop Exam 1 material

\[ S \left( 1 + x + x^2 + x^3 + \ldots \right) \, dx = \frac{c + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots}{1 - x} \]

Remember \( \int \frac{1}{x} \, dx = \ln |x| + c \) closely related