Believing in Mathematics

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In my 300-level math courses for STEM students, I assign a very challenging homework problem, an unsolved problem in math worth $1,000,000. The students know it is unsolved.

Several weeks later, students write a reflective essay about their experience with the homework in the class.

Here are some direct quotes from these essays written by junior/senior STEM majors.
Student #1: “Now that I think about it, this should probably be how I approach all the theoretical problems. Instead of trying to find the correct answer right off the bat, I should write down what I know to be true about the problem and get a better understanding of it first. Anyway, I just really enjoyed how this problem challenged me to come up with my own way of approaching the problem and how I did not feel any pressure to find the correct answer.”
Student #2: “I was quite frustrated as I did not even have a clue what to do. I found myself merely writing ‘I don’t know what to do’ in a bunch of different ways after partially starting random thoughts. I felt somewhat insignificant while rifling through my disjointed thoughts.”

Student #3: “There were moments when I was hopeful that my algebra and determination would lead me to some insight. Overall, I was unsatisfied with my work because I left feeling defeated.”
What is the message students are sending?

- Very few students discuss mathematics.
- Almost all essays focus on what students felt and what students did.

These are “successful” students in mathematics and STEM, but their underlying beliefs and practices are unproductive.

These unproductive beliefs prevent many capable students from succeeding in math courses in their first year at UK.
Modern psychology frames the human psyche as a three-stranded model.

- **Cognition**: intellectual functioning
- **Affect**: emotional functioning
- **Enaction**: behavioral functioning
In first-year mathematics courses, I need *direct interventions* to provide students with three things:

1. a language for discussion of productive struggle in the context of affective and enactive domains,
2. an environment in which such discussion arises naturally and effectively, and
3. a contemporary “external source” motivating this language and environment so that our discussion is not about the will of the instructor.

How these are realized in my classes will change over time. I am willing to follow current educational “fads” if they are effective tools for my students.
Carol Dweck’s theory of mindsets

Carol Dweck is a psychologist at Stanford who studies how beliefs influence motivation.

- The belief that each individual has an innate, fixed ability in a given area is referred to as a fixed mindset.
- The belief that individuals are capable of continually developing their abilities through persistence and effort is referred to as a growth mindset.

Growth mindsets in students play a surprisingly strong positive role in achievement, success, retention, persistence, etc, particularly for students from underrepresented groups in mathematics.

Growth/fixed mindset research is contentious among some psychologists — that is okay with me, because I only need the language and environment that this provides.
Structure of MA 114 — Calculus II

- 154 students in large lecture with me on M/W/F.
- Students attend two 75-minute recitations per week on T/Th in ~30-student sections led by a graduate student TA.
- Online homework, evening exams.
First day of class

- Begin with everyone in class introducing themselves to the other students around them.
- Next, have them spend 4-5 minutes discussing with peers a question from 5th grade that is fundamental in Calc II: Do you believe that \(0.9999999\ldots = 1\)? Why or why not? (This is a true statement.)
- I then spend a few minutes talking course structure.
- Students watch a 10-minute TEDx talk about growth and fixed mindset research.
  - Following video, students spend 2-3 minutes free response writing about the video.
  - Following the writing, students spend 2-3 minutes discussing their response with a neighbor in the class.
- After this...
Introduce the following course policy. This applies to students, TAs, and me as well.

Students are not allowed to make disparaging comments, at any time or for any reason, about themselves, their mathematical ability, their peers, or the abilities of their peers. Here are example statements that are prohibited, along with acceptable replacement phrases.

- I can’t do this → I am still learning how to do this
- That was stupid → That was a productive mistake
- This is impossible → There is something interesting and subtle in this problem
- I’m an idiot → This is going to take careful thought
- I’ll never understand this → This might take me a long time and a lot of work to figure out
- This is terrible → I think I’ve done something incorrectly, let me check it again
Key Point

We may not be geniuses, but we are all capable.
Complement Day 1 With Active Learning Techniques

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*In recitation:* Day 1, review course policy on supportive language. Day 2, watch a Veritasium video about research regarding the effectiveness of science videos and do a 2-minute free writing followed by 2-minute discussion with neighbors. Every day consists of significant amounts of small group work based on problem sets. TAs assign groups, change group assignments every 1-2 weeks to increase social interaction and introduce regular, productive disruption to learning environment.
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- My TAs report that the students are working well in groups.
- Number of visitors to my office hours is up from past semesters.
- So far, most of my students are coming to lecture.
- Two days ago one of my students called me out for violating the Course Policy during lecture.
  - I was getting frustrated with myself for making errors with inequalities in an example. After being called out, I slowly said “I’m steadily improving in my mathematical abilities with inequalities” and everyone laughed.

Hopefully this will help more of my capable students succeed.
I maintain focus on the following four “Axioms for Teaching.”

**Axiom #1:** Mathematical talent is uniformly distributed, irrespective of geographic, demographic, and economic boundaries.

**Axiom #2:** Everyone can have meaningful and rewarding mathematical experiences.

**Axiom #3:** Excellence is possible, perfection is not.

**Axiom #4:** All human systems are flawed.

Note: The first two of these axioms come from Federico Ardila, a professor at San Francisco State University.