1. Exercise 3.18.
Find all solutions in the appropriate canonical complete residue system modulo \( n \) that satisfy the following linear congruences:

1. \( 2x \equiv 3 \pmod{5} \).

2. \( 26x \equiv 14 \pmod{3} \).

3. \( 4x \equiv 7 \pmod{8} \).

4. \( 24x \equiv 123 \pmod{213} \).
2. **Theorem 3.19.**

Let $a, b,$ and $n$ be integers with $n > 0$. Show that $ax \equiv b \pmod{n}$ has a solution if and only if there exists integers $x$ and $y$ such that $ax + ny = b$.

**Proof:** Assume that $ax \equiv b \pmod{n}$ has a solution. Then there exists an integer $z$ such that $ax - b = nz$. Hence $ax + n(-z) = b$, or $ax + ny = b$ where $y = -z$. Conversely assume that there are integers $x$ and $y$ such that $ax + ny = b$. Then $ax - b = n(-y)$, which is a multiple of $n$. Thus $n|(ax - b)$ or, equivalently, $ax \equiv b \pmod{n}$.

3. **Theorem 3.20.**

Let $a, b,$ and $n$ be integers with $n > 0$. The equation $ax \equiv b \pmod{n}$ has a solution if and only if $(a, n)|b$.

**Proof:** Assume the equation $ax \equiv b \pmod{n}$ has a solution. Then by Theorem 3.19 there exist integers $x$ and $y$ such that $ax + ny = b$. Therefore by Theorem 1.48 we have that $\gcd(a, n)|b$.

Conversely, suppose that $\gcd(a, n)|b$. Then by Theorem 1.48, there exist integers $x$ and $y$ such that $ax + ny = b$. Therefore by Theorem 3.19, $ax \equiv b \pmod{n}$ has a solution.