1. (a) (5 pts) If \( f(x) = 1 - x^2 \) and \( g(x) = \sqrt{x} \) find a formula for \((g \circ f)(x)\). Give the domain of \((g \circ f)(x)\).

\[
(g \circ f)(x) = g(f(x)) = \sqrt{1-x^2}
\]

Domain: all \( x \)'s s.t. \( 1-x^2 \geq 0 \)

\[\begin{array}{c}
\text{domain: all } x \text{'s s.t. } 1-x^2 \geq 0 \\
\end{array}\]

:: domain: all \( x \)'s s.t. \[-1 \leq x \leq 1\]

(b) (5 pts) Which of the following functions are even, odd, neither? Explain your answer

1. \( f(x) = 3 + |x| - x^4 \)

\[f(-x) = 3 + |-x| - (-x)^4 = 3 + |x| - x^4 = f(x)\]

It is \underline{even}\n
2. \( g(x) = 2x^3 - x^2 + 1 \)

\[g(-1) = 2(-1)^3 - (-1)^2 + (1) = -2\]
\[g(1) = 2(1)^3 - (1)^2 + (1) = 2\]

\[
\text{but } g(2) = 2 \cdot 8 - 4 + 1 = 13
\]
\[g(-2) = 2 \cdot (-8) - 4 + 1 = -19
\]

pts: 7/10
2. Compute the following limits. Each limit is worth 5 points.

(a) \[
\lim_{x \to -1} \frac{x^2 - 3x - 4}{x + 1} = \lim_{x \to -1} (x - 4) = 0
\]

(b) \[
\lim_{h \to 0} \frac{1}{h} [(h - 4)^2 - 16] = \lim_{h \to 0} \frac{h - 8}{h} = 0
\]

(c) \[
\lim_{x \to 6^+} \frac{(x - 5)(3 - x)}{(x - 6)(x - 1)} = \lim_{x \to 6^-} \frac{(x - 5)(3 - x)}{(x - 6)(x - 1)} = \frac{1 \cdot (-3)}{0^+ \cdot 5} = -\infty,
\]

(d) \[
\lim_{x \to 2} \frac{x^4 - 16}{x - 2} = \lim_{x \to 2} (x + 2)(x^2 + 4) = (2 + 2)(2^2 + 4) = 32
\]

(e) Find \( c = 11 \) so that \[
\lim_{x \to 1} \frac{x^2 + cx - x - c}{x^2 + 2x - 3} = \frac{3}{4}
\]

\[
\frac{1 + c}{4} = 3 \quad \therefore \quad c = 11
\]

\( \text{pts: } 7/25 \)
3. Find all the values of the constant $c$ that make the function

$$h(x) = \begin{cases} 
  c^2 - x^2 & \text{if } x < 1 \\
  2(x-c)^2 & \text{if } x \geq 1
\end{cases}$$

continuous everywhere. Graph these functions.

The only problem is at $x=1$. We want to make sure that

$$\lim_{x \to 1^-} h(x) = \lim_{x \to 1^+} h(x) \quad \text{But this means that}$$

$$c^2 - 1 = 2(1-c)^2 \quad \text{or}$$

$$2 - 4c + 2c^2 - c^2 + 1 = 0 \quad \therefore c^2 - 4c + 3 = 0 \quad \therefore (c-3)(c-1) = 0$$

$$\therefore c = 1, \ c = 3$$

\[ h_1(x) = \begin{cases} 
  1 - x^2 & x \leq 1 \\
  2(x-1)^2 & x \geq 1
\end{cases} \]

\[ h_2(x) = \begin{cases} 
  9 - x^2 & x \leq 1 \\
  2(x-3)^2 & x \geq 1
\end{cases} \]
4. Does the equation \( x^3 + 3x - 2 = 0 \) have a root between 0 and 1. Explain. 
(Note: A calculator solution is not an acceptable answer.)

Use the Intermediate value theorem!
\[
f(x) = x^3 + 3x - 2 \text{ is continuous on } [0, 1] \\
\text{and } f(0) = -2 \text{ while } f(1) = 1^3 + 3 \cdot 1 - 2 = 0
\]

\[
\begin{array}{c}
\text{the graph of } f(x) \text{ intersects the } x-\text{axis between } 0 \text{ and } 1.
\end{array}
\]

Thus, there is a root.

pts: 7/7

5. A segment of the tangent line to the graph of \( f(x) \) at \( x = 2 \) is shown in the diagram. Using information from the graph we can estimate that

\[
f(2) = 0 \quad f'(2) = -2
\]

Hence the equation of the tangent line to the graph of

\[
g(x) = 5x + f(x)
\]

at \( x = 2 \) is \( y = 3x + 4 \)

\[
P(2, g(2)) = (2, 10)
\]

as \( g(2) = 5 \cdot 2 + f(2) = 10 + 0 = 10 \)

\[
m = g'(2) \quad \text{But} \quad g'(x) = 5 + f'(x)
\]

so \( g'(2) = 5 + (-2) = 3 \)

\[
\therefore y - 10 = 3(x - 2)
\]

pts: 7/8
6. Calculate the following derivatives. Each derivative is worth 5 points.

Do not simplify your answers.

(a) If \( f(x) = 3x^2 - \frac{x}{\pi} + \pi^2 \) then \( f'(x) = \frac{6x - 1}{\pi} \).

No h e c : \frac{1}{\pi}, \frac{2}{\pi} are numbers ! ! !

(b) If \( f(x) = (x^3 - 3)(-3x - x^2) \) then \( f'(x) = \frac{3x^2 \cdot (-3x - x^2) + (x^3 - 3)(-3 - 2x)}{(x + 1)^2} \).

(c) If \( g(t) = \frac{2t - 1}{t + 1} \) then \( g'(t) = \frac{2 \cdot (t + 1) - (2t - 1) \cdot 1}{(t + 1)^2} \).

(d) If \( p(t) = t^{3/2} - t^{-1/2} - 3 \) then \( p'(t) = \frac{3}{2} t^{1/2} + \frac{1}{2} t^{-3/2} \).

pts: /20
7. Match the graph of each function labelled (a)-(f) with the graph of its derivative (1)-(6).
8. A ball is thrown upward at 64 feet per second from a height of 80 feet. In the absence of air resistance it will have height

\[ h(t) = -16t^2 + 64t + 80 \text{ feet.} \]

(a) (3 pts) After how many seconds will the ball hit the ground?

\[ h(t) = 0 \]

\[-16t^2 + 64t + 80 = 0 \Rightarrow t^2 - 4t - 5 = 0 \]

\[(t - 5)(t + 1) = 0 \quad \therefore (t = 5) \quad t = -1\]

(b) (2 pts) What will the velocity of the ball be 2 seconds after it is thrown?

\[ v(t) = h'(t) = -32t + 64 \]

\[ v(2) = -32 \cdot 2 + 64 = 0 \text{ ft/s} \]

\[ \therefore t = 2 \text{ corresponds to the max of the parabola} \]

(c) (2 pts) What will the velocity of the ball be when it hits the ground?

\[ \Rightarrow v(5) = -32 \cdot 5 + 64 = -160 + 64 = -96 \text{ ft/s} \]

from part (a), \( t = 5 \)

(d) (3 pts) How high will the ball go?

from part (b) the highest point occurs when \( t = 2 \).

\[ h(2) = -16 \cdot 2^2 + 64 \cdot 2 + 80 \]

\[ = \ldots = 144 \text{ feet} \]