Ma123 First Examination
September 24, 2003
Instructions:

There are 9 problems on 7 pages (including this cover page). Each problem is worth 10 points.

Check that you have a complete exam.

Show all work and explain your answers. Unsupported answers will receive no credit.

Fill in the information below and put your name or initials on each of the other pages.

Name: ____________________________

Instructor: __________________________

Section Number/Class Meeting Time: ____________

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[ Homework Percentage ]

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1
Problem 1. Do the following calculations:

\[ \frac{1}{2} \cdot \frac{1}{2} = \left( \frac{1}{\sqrt{100}} \right)^3 = \frac{1}{1000} \]

a. Given \( f(x) = x^2 \), what is \( f(100) \)?

b. Find the points of intersection of the graphs of \( y = x^2 - 3 \) and \( y = 2x \)

\[
\begin{align*}
\begin{cases}
  y &= x^2 - 3 \\
  y &= 2x
\end{cases} & \implies 2x = x^2 - 3 \\
\end{align*}
\]

\[ x^2 - 2x - 3 = 0 \quad \iff (x-3)(x+1) \]

\[ x = 3, \quad x = -1 \]

Problem 2: Let \( f(x) = 2x^3 - 7x - 7 \).

The equation of the tangent line to the graph of \( f(x) \) at \( x = -1 \) is \( y = \ldots x + \ldots \)

\[
\begin{align*}
  f'(x) &= 6x^2 - 7 \\
  f'(-1) &= 6 - 7 = -1
\end{align*}
\]

\[
\begin{align*}
  y - (-2) &= -1(x - (-1)) \\
  y &= -x - 1 - 2 \\
  y &= -x - 3
\end{align*}
\]
Problem 3: Determine which of the following limits exist. Calculate the exact value of any that do exist.

a. \( \lim_{x \to 9} \frac{\sqrt{x^2 - 5x - 36}}{8 - 3x} = \frac{\sqrt{81 - 45 - 36}}{8 - 27} = \frac{0}{-19} = 0 \)

b. \( \lim_{x \to \infty} \frac{1}{|x| - 1} = \frac{1}{\infty} = 0 \)

Problem 4: What is the value of \( x \) for which the tangent line to the graph of \( f(x) = 3x^2 - 11x \) is parallel to the line whose equation is \( y = x \)?

\[
\begin{align*}
\frac{d}{dx} f(x) &= 3x^2 - 11x \\
\Rightarrow f'(x) &= 6x - 11
\end{align*}
\]

\( \text{Want} \)

\( f'(x) = 1 \)

\[
\begin{align*}
6x - 11 &= 1 \\
6x &= 12 \\
\therefore x &= 2
\end{align*}
\]
Problem 5: Determine the unknowns $A$ in each of the following:

a. If \( \lim_{{x \to 0}} \frac{3x^7 + 6x^5 + 6A}{x^7 - 2x^4 - 2} = 2 \), then $A = \_\_\_\_\_\_\_\_\_$.

\[
\lim_{{x \to 0}} \frac{3x^7 + 6x^5 + 6A}{x^7 - 2x^4 - 2} = \frac{6A}{-2} = 2
\]

\[A = -\frac{4}{6} = -\frac{2}{3}\]

b. If \( \lim_{{x \to \infty}} \frac{Ax^7 + 6x^5 + 6}{x^7 - 2x^4 - 2} = 2 \), then $A = \_\_\_\_\_\_\_\_\_\_\_\_\_$.

\[\lim_{{x \to \infty}} \frac{Ax^7 + 6x^5 + 6}{x^7 - 2x^4 - 2} = \frac{A}{1} = 2 \quad \therefore \quad [A = 2]\]

c. If $f(x) = \begin{cases} 
A + 2x & \text{for } x \leq 3 \\
A & \text{for } 3 < x
\end{cases}$ is continuous then $A = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$.

\[
\int f(3) = A + 6 = \lim_{{x \to 3^+}} f(x) = 3 - A
\]

\[\therefore \quad 2A = 3 - 6 \quad \therefore \quad [A = -\frac{3}{2}]\]
Problem 6: Calculate the following. DO NOT NEED TO SIMPLIFY YOUR ANSWERS.

a. Let \( f(x) = x^{\frac{1}{3}} \) then \( f'(8) = \frac{1}{12} \).  

\[
    f'(x) = \frac{1}{3} x^{-\frac{2}{3}} = \frac{1}{3} x^{\frac{2}{3}} \quad f'(8) = \frac{1}{3} \left( \frac{3}{8} \right)^{\frac{2}{3}}
\]

b. Let \( f(x) = x^3 + 5x^2 + 1 \). Then \( f'(x) = \frac{2x^2 + 10x}{3} \).

c. Let \( f(x) = \frac{1}{x^2 + 4} \). Then \( f'(2) = \frac{1}{12} \).

d. Let \( f(x) = (2x - 1)(x - 5) \) then \( f'(0) = \frac{11}{2} \).
Problem 7: Let \( f(x) = \begin{cases} 
3 & x \leq 1 \\
x+2 & 1 < x < 2 \\
-2 & 2 \leq x 
\end{cases} \)

a. Sketch the graph of \( f(x) \) on the graph paper below.

b. Is \( f(x) \) **continuous** at \( x = 1 \)?
Explain your answer. Your answer counts one point, your explanation counts 2.

\[
\text{Yes} \quad f(1) = 3 = \lim_{x \to 1} f(x)
\]

c. Is \( f(x) \) **differentiable** at \( x = 2 \)?
Explain your answer. Your answer counts one point, your explanation counts 2.

\[
\text{No} \quad \text{It is not even continuous.}
\]
Problem 8: If \( f(x) = 3x^2 + 6x + 6 \), use the (limit) definition to calculate \( f'(x) \).

Note: you must use the limit definition. Simply using the derivative rules to get the answer will earn no credit.

\[
\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{3(x+h)^2 + 6(x+h) + 6 - 3x^2 - 6x - 6}{h}
\]

\[
= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 + 6x + 6h + 6 - 3x^2 - 6x - 6}{h}
\]

\[
= \lim_{h \to 0} \frac{6x + 3h + 6}{h} = \frac{6x + 6}{h}
\]

Problem 9: Suppose \( f \) and \( g \) are functions. Some values of \( f, f', g, \) and \( g' \) are given in the table.

\[
\begin{array}{c|cccc}
 x & f(x) & f'(x) & g(x) & g'(x) \\
\hline
2 & 0 & 1 & 4 & -1 \\
4 & 2 & 3 & -2 & -4 \\
\end{array}
\]

a. If \( h(x) = \frac{f(x)}{g(x)} \) then \( h'(2) = \)

b. If \( h(x) = f(x)g(x) \) then \( h'(2) = \)

c. If \( h(x) = f(x) + xg(x) \) then \( h'(4) = \)