Newton's Approximation of Pi

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Outline

• Who was Isaac Newton? What was his life like?

• What is the history of Pi?

• What was Newton’s approximation of Pi?
History of Isaac Newton

- 17th Century

  - Shift of progress in math

  - “relative freedom” of thought in Northern Europe
The Life of Newton

- Born: Christmas day 1642
- Died: 1727
- Raised by grandmother
Newton’s Education

• 1661
• Began at Trinity College of Cambridge University

• 1660
• Charles II became King of England
• Suspicion and hostility towards Cambridge
Newton, the young man

• “single minded”
  - Would not eat or sleep over an intriguing problem

• Puritan
  - Book of sins
Newton's Studies

• 1664
  - Promoted to scholar at Trinity

• 1665-1666
  - Plague
  - Newton’s most productive years
Newton’s Discoveries

• 1665
  - Newton’s “generalized binomial theorem”
  - led to method of fluxions

• 1666
  - Inverse method of fluxions
  - Began observations of rotation of planets
Newton’s Accomplishments

• 1668
  - Finished master’s degree
  - Elected fellow of Trinity College

• 1669
  - Appointed Lucasian chair of mathematics
Newton's Accomplishments

• @ 1704
  - Elected President of the Royal Society

• 1705
  - Knighted by Queen Anne

• 1727
  - Buried in Westminster Abbey
The History of Pi

• Archimedes' classical method
  - Using Polygons with inscribed and circumscribed circles
- Found Pi between 223/71 and 22/7
  • ≈ 3.14
Important Dates of Pi

• 150 AD
  - First notable value for Pi by Caludius Ptolemy of Alexandria
  - \( \pi = 3 \ 8'30'' \)
  - \( = \frac{377}{120} \)
  - \( = 3.1416 \)
• 480 AD
  - TSU Ch’ung-chih from China gave rational approximation
  - $\pi = \frac{355}{113}$
    = 3.1415929

• 530 AD
  - Hindu mathematician Aryabhata
  - $\pi = \frac{62,832}{20,000}$
    = 3.1416
• 1150 AD
  - Bhaskara
  - $\pi = 3\frac{927}{1250}$
  - $\pi = \frac{22}{7}$
  - $\pi = \frac{754}{240}$
  - $\approx 3.1416$
• 1429 AD
  - Al- Kashi
  - Astronomer approximated Pi to 16 decimal places

• 1579 AD
  - Francois Viete from France
  - Approximated Pi to 9 decimal places
• 1585 AD
  - Adriaen Anthoniszoon
  - Rediscovered Chinese ratio 355/113
  - $377/120 > \pi > 333/106$

• 1593 AD
  - Adriaen Von Roomen
  - Found $\pi$ to the 15th decimal place by classical method using polygons with $2^{30}$th sides
• 1610 AD
  - Ludolph Van Ceulen of the Netherlands
  - Pi ~ 30 decimal places
  - Used polygons with $2^{62}$ sides
• 1621 AD
  - Willebrord Snell (Dutch)
  - Able to get Ceulen’s 35$^{th}$ decimal place by only $2^{30}$ side polygon
• 1630 AD
  - Grienberger
  - Pi to 39 decimal places

• 1671
  - James Gregory from Scotland obtained infinite series

\[
\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots (-1 \leq x \leq 1)
\]
• 1699 AD
  - Abraham Sharp
  - Pi ~ 71 decimal places
• 1706 AD
  - John Machin
  - Pi ~ 100^{th} decimal place
• 1719 AD
  - De Lagny of France
  - Pi ~ 112 decimal places

• 1737 AD
  - William Jones from England
  - First to use Pi symbol for ratio of the circumference to the diameter
• 1767 AD
  - Johan Heinrich Lambert
  - Showed Pi is irrational
• 1794 AD
  - Adrien-Marie Legendre
  - Showed Pi-squared is irrational
• 1841 AD
  - William Rutherford
  - Calculated Pi to 208 places

• 1844 AD
  - Zacharis Dase found Pi correct to 200 places using Gregory Series

\[
\frac{\pi}{2} = \arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{5}\right) + \arctan\left(\frac{1}{8}\right)
\]
• 1853 AD
  - Rutherford returns
  - Finds Pi to 400 decimal places

• 1873 AD
  - William Shanks from England
  - Pi to 707 decimal places

• 1882 AD
  - F. Lindeman
  - Shows Pi is transcendental
• 1948
  - D.F. Ferguson of England
    • Finds errors with Shanks value of Pi starting with the 528\(^{th}\) decimal place
    • Gives correct value to the 710\(^{th}\) place
  - J.W. Wrench Jr.
    • Works with Ferguson to find 808\(^{th}\) place for Pi
    
    Used Machin’s formula
    
    \[
    \frac{\pi}{4} = 3 \arctan\left(\frac{1}{4}\right) + \arctan\left(\frac{1}{20}\right) + \arctan\left(\frac{1}{1985}\right)
    \]
• 1949 AD
  - Electronic computer - The ENIAC
    - Compute Pi to the 2,037\textsuperscript{th} decimal places

• 1959 AD
  - Fancois Genuys from Paris
  - Compute Pi to 16,167 decimal places with IBM 704
• 1961 AD
  - Wrench and Shanks of Washington D.C.
  - compute Pi to 100,265\textsuperscript{th}
    using IBM 7090

• 1966 AD
  - M. Jean Guilloud and co-workers
  - attained approximation for Pi
    to 250,000 decimal places on a STRETCH computer
• 1967 AD  
  - M. Jean Guilloud and coworkers  
  - found Pi to the 500,000 places on a CDC 6600

• 1973  
  - M. Jean Guilloud and coworkers found Pi to 1 millionth place on CDC 7600

• 1981 AD  
  - Kazunori Miyoshi and Kazuhika Nakayma of the University of Tsukuba  
  - Pi to 2 million and 38 decimal places in 137.30 hours on a FACOM M-200 computer
• 1986 AD
  - DH Bailey of NASA Ames Research Center ran a Cray-2 supercomputer for 28 hours
    • Got Pi to 29,360,000 decimal places
  - Yasamasa Kanada from University of Tokyo
    • Used NEC SX-2 super computer to compute Pi to 134,217,700 decimal places
Purpose to Continue to Compute Pi

- See if digits of Pi start to repeat
  - Possible normalcy of Pi
- Valuable in computer science for designing programs
Information Already known

\[
\left(x - \frac{1}{2}\right)^2 + (y - 0)^2 = \frac{1}{2}
\]

or

\[
x^2 - x + \frac{1}{4} + y^2 = \frac{1}{4}
\]
Solve for “y”

\[ y = x^{1/2} (1 - x)^{1/2} \]

\[ = x^{1/2} \left( 1 - \frac{1}{2} x - \frac{1}{8} x^2 - \frac{1}{16} x^3 - \frac{5}{128} x^4 - \frac{7}{256} x^5 - \ldots \right) \]

\[ = x^{1/2} - \frac{1}{2} x^{3/2} - \frac{1}{8} x^{5/2} - \frac{1}{16} x^{7/2} - \frac{5}{128} x^{9/2} - \frac{7}{256} x^{11/2} - \ldots \]
Area (ABD) by fluxion

\[
\frac{2}{3} x^{3/2} - \frac{1}{2} \left( \frac{2}{5} x^{5/2} \right) - \frac{1}{8} \left( \frac{2}{7} x^{7/2} \right) - \frac{1}{16} \left( \frac{2}{9} x^{9/2} \right) - \ldots
\]

\[
= \frac{2}{3} x^{3/2} - \frac{1}{5} x^{5/2} - \frac{1}{28} x^{7/2} - \frac{1}{72} x^{9/2} - \frac{5}{704} x^{11/2} - \ldots
\]
\[
\left( \frac{1}{4} \right)^{3/2} = \left( \sqrt[3]{\frac{1}{4}} \right)^3 = \frac{1}{8}, \quad \left( \frac{1}{4} \right)^{5/2} = \left( \sqrt[5]{\frac{1}{4}} \right)^5 = \frac{1}{32} \ldots
\]

\[
\frac{1}{12} - \frac{1}{160} - \frac{1}{3584} - \frac{5}{1441792} \ldots - \frac{429}{163208757248} = .07677310678
\]
Area (ABD) by geometry

\[ BD = \sqrt{\left(\frac{1}{2}\right)^2 - \left(\frac{1}{4}\right)^2} = \sqrt{\frac{3}{16}} = \frac{\sqrt{3}}{4} \]

\[ \text{Area}(\triangle DBC) = \frac{1}{2} (BC) \times (BD) = \frac{1}{2} \left(\frac{1}{4}\right) \left(\frac{\sqrt{3}}{4}\right) = \frac{\sqrt{3}}{32} \]
Area(sector) = \frac{1}{3} \text{Area(semicircle)}

= \frac{1}{3} \left( \frac{1}{2} \cdot \pi \cdot r^2 \right)

= \frac{1}{3} \left[ \frac{1}{2} \pi \left( \frac{1}{2} \right)^2 \right]

= \frac{\pi}{24}
Area(ABD) = Area(sector) − Area(ΔDBC)

= \frac{\pi}{24} − \frac{\sqrt{3}}{32}

π ≈ 24 \left(0.07677310678 + \frac{\sqrt{3}}{32}\right) = 3.141592668...