Review Problems for Midterm I, MA213, Fall 2013

Exam Date: Wednesday, October 9 2013

- The time of exam is: 2:00-2:50 pm.
- The place of exam is: CB 106

Here is a set of review problems.
1. Compute the distance between \( \mathbf{A} = (2, -1, 7) \) and \( \mathbf{B} = (1, -3, 5) \).

2. Calculate the angle between the two vectors
   \[ \mathbf{a} = (2, -3, 1), \quad \mathbf{b} = (1, 6 - 2). \]

3. Calculate the area of the parallelogram formed by the two edges \( \mathbf{AB} \) and \( \mathbf{AC} \), where
   \[ \mathbf{A} = (1, 3, 4), \quad \mathbf{B} = (2, 7, -5), \quad \mathbf{C} = (-1, 0, 3). \]

4. Calculate the volume of the parallelepiped formed by the adjacent edges \( \mathbf{PR}, \mathbf{PQ}, \mathbf{PS} \), where
   \[ \mathbf{P} = (2, 0, -1), \quad \mathbf{Q} = (4, 1, 0), \quad \mathbf{R} = (3, -1, 1), \quad \mathbf{S} = (2, -2, 2). \]

5. Calculate the distance from the point \( \mathbf{P} = (1, 3, 2) \) to the plane
   \[ 4x + 5y + 6z + 7 = 0. \]

6. Find the equation of the line passing through two points
   \[ \mathbf{P} = (1, 2, 3), \quad \mathbf{Q} = (-2, -1, -3). \]

7. Find the equation of the plane passing through two lines:
   L1: \( x = t + 1, \ y = 2t + 2, \ z = 3t + 3, \)
   L2: \( x = -2t + 4, \ y = 3t - 3, \ z = -t. \)

8. a) Suppose the rectangle coordinate of a point \( \mathbf{P} \) is \( (1, \sqrt{3}, 2\sqrt{3}) \). Calculate
both its cylindrical coordinate and spherical coordinate.
b) Suppose the spherical ordinate of $Q$ is $(2\sqrt{2}, \frac{\pi}{4}, \frac{\pi}{3})$. Find its rectangle coordinate and cylindrical coordinate.

9. 
a) Calculate the velocity and acceleration of a particle whose position function is 
   \[ \mathbf{r}(t) = (t, t^2, t^3). \]

b) Find the position function of a particle if its initial position is $\mathbf{r}(0) = (1, -1, 2)$, initial velocity is $\mathbf{v}(0) = (-2, 0, 3)$, and the acceleration is 
   \[ \mathbf{a}(t) = (-2t, 3t^2, e^t). \]

10. Calculate the arc length of the curve 
    \[ \mathbf{r}(t) = (e^t, e^t \sin t, e^t \cos t), \quad 0 \leq t \leq 2\pi. \]

11. Calculate the curvature of the curve 
    \[ \mathbf{r}(t) = (e^{-t}, 2t, \ln(1 + t^2)). \]

12. Calculate the unit tangent vector $\mathbf{T}$ and the normal vector $\mathbf{N}$ of the curve 
    \[ \mathbf{r}(t) = (\sqrt{2} \cos t, \sin t, \sin t). \]

13. Let $f(x, y) = \frac{x^3 + y^3}{x^2 + y^2}$ for $(x, y) \neq (0, 0)$. Find \( \lim_{(x,y) \to (0,0)} f(x,y) \).

14. Find the first order partial derivatives of $f(x, y) = \frac{x}{\sqrt{x^2 + y^2}}$ for $(x, y) \neq (0, 0)$.

15. Find the linear approximation of $f(x, y) = \sqrt{9 - 4(x^2 + y^2)}$ at $(x, y) = (1, 1)$ and use it to approximate $f(1.01, 0.09)$.

16. Estimate $f(1.02, 0.01, -0.03)$ assuming that 
    \[ f(1, 0, 0) = -3, \quad f_x(1, 0, 0) = -2, \quad f_y(1, 0, 0) = 4, \quad f_z(1, 0, 0) = 2. \]