MA 114-Honors Worksheet # 14: Integration by Parts and Trigonometric Integrals

1. Use the product rule to find \((u(x)v(x))'\). Next use this result to prove integration by parts, namely that \(\int u(x)v'(x)dx = u(x)v(x) - \int v(x)u'(x)dx\).

2. Which of the following integrals should be solved using substitution and which should be solved using integration by parts?
   
   (a) \(\int x \cos(x^2) \, dx\),  
   (b) \(\int e^x \sin(x) \, dx\),  
   (c) \(\int \frac{\ln(\arctan(x))}{1 + x^2} \, dx\),  
   (d) \(\int xe^{x^2} \, dx\)

   Using these examples, try and formulate a general rule for when integration by parts should be used as opposed to substitution.

3. Solve the following integrals using integration by parts:
   
   (a) \(\int x^2 \sin(x) \, dx\),  
   (b) \(\int (2x + 1)e^x \, dx\),  
   (c) \(\int x \sin(3 - x) \, dx\),  
   (d) \(\int 2x \arctan(x) \, dx\),  
   (e) \(\int \ln(x) \, dx\)

4. Prove the reduction formula \(\int x^n e^x \, dx = x^n e^x - n \int x^{n-1} e^x \, dx\). Use this to evaluate \(\int x^3 e^x \, dx\).

5. Let \(f(x)\) be a twice differentiable function with \(f(0) = 6\), \(f(1) = 5\), and \(f'(1) = 2\). Evaluate \(\int_0^1 xf''(x) \, dx\).

6. Evaluate the following integrals.
   
   (a) \(\int \cos^2(x) \, dx\),  
   (b) \(\int_0^{\pi/2} \sin^2(x) \cos^2(x) \, dx\),  
   (c) \(\int \sin^3(x) \cos^2(x) \, dx\),  
   (d) \(\int x^2 \cos(x) \, dx\),  
   (e) \(\int e^x \cos(x) \, dx\)

7. Evaluate \(\int \sin(x) \cos(x) \, dx\) by four methods
   
   (a) the substitution \(u = \cos(x)\),  
   (b) the substitution \(u = \sin(x)\),  
   (c) the identity \(\sin(2x) = 2 \sin(x) \cos(x)\),  
   (d) integration by parts.

8. Find the volume of the solid obtained by rotating \(f(x) = e^x\) about the y-axis from \(0 \leq x \leq 2\).