1. Consider the following matrices.

\[ A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}, \quad D = \begin{pmatrix} 4 & -2 \\ 2 & -1 \end{pmatrix}. \]

(a) Compute the matrices \( AB \) and \( BA \).
(b) Compute the matrices \( CD \) and \( DC \).

2. (Diagonal matrices) A square matrix is called a diagonal matrix if all entries off of the main diagonal are 0.

(a) Let \( A \) be any \( n \times 2 \) matrix and let \( B = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \). Describe the columns of the matrix \( AB \) in terms of the columns of \( A \).
(b) Generalize part (a) to describe the columns of \( AB \) if \( A \) is \( n \times k \) and \( B \) is the diagonal matrix

\[ \begin{pmatrix} b_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & b_k \end{pmatrix}. \]
(c) Let \( A \) be any \( 2 \times n \) matrix and let \( B = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \). Describe the rows of the matrix \( BA \) in terms of the rows of \( A \).
(d) Generalize part (c) to describe the rows of \( BA \) if \( A \) is \( n \times k \) and \( B \) is the diagonal matrix

\[ \begin{pmatrix} b_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & b_n \end{pmatrix}. \]

3. (Scalar matrices) A diagonal matrix in which all of the diagonal entries are the same is called a scalar matrix. Use the previous problem to show that if \( A \) is any \( n \times n \) matrix and \( D \) is an \( n \times n \) scalar matrix, then \( AD = DA \). What is another description of the matrix \( AD \)?

4. One of the following three matrices has an inverse. Which one?

\[ A = \begin{pmatrix} 1 & 3 & -2 \\ 1 & 1 & -2 \\ -2 & 7 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 3 & -2 \\ 0 & 1 & -1 \\ -4 & 0 & 6 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & 0 & -3 \\ 1 & 6 & -2 \\ -4 & 0 & 6 \end{pmatrix}. \]

Hint: You shouldn’t need to do any row reducing to eliminate two possibilities.