1. Let $D$ be the region in the plane bounded by $x \geq 0$, $y \geq 0$, and the ellipse $4x^2 + y^2 = 4$ and let $R$ be the solid lying above $D$ and below the graph of the function $f(x, y) = x + 2y$.

(a) Set up, but do not evaluate, the two double integrals in rectangular coordinates that calculate the volume of $R$. (3 points)

**SOLUTION:**

$$
\int_0^2 \int_0^{\sqrt{4 - y^2}/2} x + 2y \, dx \, dy \\
\int_0^1 \int_0^{2\sqrt{1 - x^2}} x + 2y \, dy \, dx
$$

(b) Set up, but do not evaluate, an integral in polar coordinates that calculates the volume of $R$. (2 points)

**SOLUTION:**

Converting the equation $4x^2 + y^2 = 4$ to polar we obtain $r^2(4 \cos^2 \theta + \sin^2 \theta) = 4$ or $r^2(3 \cos^2 \theta + 1) = 4$. Solving for $r$ we have $r = \frac{2}{\sqrt{1 + 3 \cos^2 \theta}}$. Thus remembering the factor of $r$ that comes in when we convert to polar we have:

$$
\int \int_D x + 2y \, dA = \int_0^{\pi/2} \int_0^{2/\sqrt{1 + 3 \cos^2 \theta}} (r \cos \theta + 2r \sin \theta) r \, dr \, d\theta
$$
2. Let $D$ be the region in the plane bounded below by the x-axis and above by the circle $x^2 + y^2 = 1$. Convert the double integral $\iint_D x + 2 \, dA$ into polar coordinates and evaluate the double integral. (3 points)

**SOLUTION:**

\[
\int \int_D x + 2 \, dA = \int_0^\pi \int_0^1 (r \cos \theta + 2) r \, dr \, d\theta = \pi
\]

3. Rewrite the given triple integral in the two orders $dx \, dy \, dz$ and $dz \, dx \, dy$:

\[
\int_0^1 \int_0^{\sqrt{x}} \int_0^{1-y} f(x, y, z) \, dz \, dy \, dx.
\]

(2 points)

**SOLUTION:**

\[
\int_0^1 \int_0^{1-y} f(x, y, z) \, dz \, dy \, dx = \int_0^1 \int_0^1 \int_0^{y^2} f(x, y, z) \, dx \, dy \, dz
\]

\[
\int_0^1 \int_0^{1-y} f(x, y, z) \, dz \, dy \, dx = \int_0^1 \int_0^{y^2} \int_0^{1-y} f(x, y, z) \, dx \, dy \, dz
\]