MA 114 Worksheet # 4: Summing an Infinite Series

1. (Review) Compute the following sums
   
   (a) \[ \sum_{n=1}^{5} 3n \]
   
   (b) \[ \sum_{k=3}^{6} \left( \sin \left( \frac{\pi}{2} + \pi k \right) + 2k \right) \]

2. Conceptual Understanding:
   
   (a) What is a series?
   
   (b) What is the difference between a sequence and a series?
   
   (c) What does it mean that a series converges?

3. Write the following in summation notation:
   
   (a) \( \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36} + \ldots \)
   
   (b) \( 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots \)

4. Calculate \( S_3 \), \( S_4 \), and \( S_5 \) and then find the sum of the telescoping series \( S = \sum_{n=1}^{\infty} \left( \frac{1}{n+1} - \frac{1}{n+2} \right) \).

5. Use Theorem 3 of 10.2 (Divergence Test) to prove that the following two series diverge:
   
   (a) \[ \sum_{n=1}^{\infty} \frac{n}{10n + 12} \]
   
   (b) \[ \sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2 + 1}} \]

6. Use the formula for the sum of a geometric series to find the sum or state that the series diverges and why:
   
   (a) \( \frac{1}{1} + \frac{1}{8} + \frac{1}{8^2} + \ldots \)
   
   (b) \[ \sum_{n=0}^{\infty} \left( \frac{\pi}{e} \right)^n \]
   
   (c) \( 5 - \frac{5}{4} + \frac{5}{4^2} - \frac{5}{4^3} + \ldots \)
   
   (d) \[ \sum_{n=0}^{\infty} \frac{8 + 2^n}{5^n} \]