1. Prove that the circle $S^1$ is a differentiable manifold using the stereographic projection.

2. Complete the proof (sketched on page 3 of do Carmo) that a differentiable manifold is a topological space, that the sets $x_\alpha(U_\alpha)$ are open, and that the maps $x_\alpha$ are continuous.

3. Suppose that $\gamma : I \subset \mathbb{R} \rightarrow \mathbb{R}^n$ is a differentiable curve defined for the interval $I$, and that $f : U \subset \mathbb{R}^n \rightarrow \mathbb{R}$ is a $C^1$-function defined on an open set $U$ containing $\gamma(I)$. Show that for any $t \in I$,

$$\frac{d}{dt} (f \circ \gamma)(t) = (\nabla f)(\gamma(t)) \cdot \gamma'(t),$$

where $\nabla f$ is the gradient of $f$ and $\gamma'(t)$ is the tangent vector to the curve $\gamma$ at $t$.

4. The sphere $S^{n-1} \subset \mathbb{R}^n$ can be realized as the zero set of the function

$$F(x_1, \ldots, x_n) = \sum_{i=1}^{n} x_i^2 - 1.$$ 

Suppose that $\gamma : I \subset \mathbb{R} \rightarrow \mathbb{R}^n$ is a differentiable curve defined for the interval $I$ that obeys the equation

$$F(\gamma(t)) = 0, \ \forall t \in I.$$ 

Prove that

$$\gamma(t) \cdot \gamma'(t) = 0, \ \forall t \in I.$$ 

What does this condition mean geometrically?