MA162: Finite mathematics

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University of Kentucky

April 2, 2012

Schedule:

- HW 6.2, 6.3 are due Fri, April 6th, 2012
- Exam 3 is Monday, Apr 9th, 5:00pm-7:00pm in CB106 and CB118.

Today we will cover 6.3: Multiplication principle
Exam 3 breakdown

- Chapter 5, Interest and the Time Value of Money
  - Simple interest
  - Compound interest
  - Sinking funds
  - Amortized loans
- Chapter 6, Counting
  - Inclusion exclusion
  - Multiplication principle
  - Permutations and combinations
How many squares in this figure?

Each column has 3 squares, there are 7 columns, so $3 \times 7 = 21$.
Counting each square is slower and error-prone.
6.3: What is multiplication?

- How many squares in this figure?

![Diagram of a grid with 7 columns of 3 squares each]

- Each column has 3 squares, there are 7 columns, so $3 \cdot 7 = 21$
6.3: What is multiplication?

- How many squares in this figure?

- Each column has 3 squares, there are 7 columns, so $3 \cdot 7 = 21$

- Counting each square is slower and error-prone.
6.3: Three square meals a day

- You decide to brush your teeth after every meal, but are worried about the toothpaste consumption. You use about 1% of the tube every time you brush. How many weeks will it last?

So 21 brushes per week; takes less than 5 weeks to use up a tube.
6.3: Three square meals a day

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- How many brushes per week?

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So 21 brushes per week; takes less than 5 weeks to use up a tube.
You are working on a dazzling fashion project and have seven dyes: Red, Orange, Yellow, Green, Blue, Indigo, and Violet. You’ve got three types of fabric: Burlap, Cotton, and Denim.

How many different color/texture combinations do you have?
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How many different color/texture combinations do you have?

Again $3 \times 7 = 21$
6.3: Counting with no overlaps

- Suppose you want to go watch a movie; you could go see one of the 12 movies at the huge theater or one of the 2 movies at the Kentucky. How many possibilities are there?

\[12 + 2 = 14\]

- Suppose you want to do a critical comparison of Hollywood with low budget art film, so you plan on going to one movie at each theater. How many possibilities are there?

\[(12)(2) = 24\]

- Suppose you are doing a study on primacy and its effect on critical comparisons, so you need to convince a bunch of your film critic friends to go see a movie at each theater, but you care which theater they go to first. How many possibilities are there?

\[(12)(2)(2) = 48\]
6.3: Counting with no overlaps

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\[ (12)(2)(2) = 48 \]
If you roll a red die and a blue die, how many possible outcomes are there?
6.3: Flipping out

- If you roll a **red die** and a **blue die**, how many possible outcomes are there?

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- A picture is easier: 36 ways
If you roll a **red die** and a **blue die**, how many possible outcomes are there?

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A picture is easier: 36 ways

Get a **penny**, a **nickel**, and a **dime**. Flip all three.

How many possibilities?
6.3: Flipping out

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- A picture is easier: 36 ways

- Get a **penny**, a **nickel**, and a **dime**. Flip all three.

  How many possibilities?

- **HHH, HHT, HTH, HTT, THH, THT, TTH, TTT**

  \((2)(2)(2) = 8\)
There are two main ways to get to Winchester from Lexington: Winchester Rd (US-60) and I-64.
From Winchester, there are three main ways to Clay City: KY-89, KY-15, and the Mountain Parkway (KY-402).
How many different ways are there from Lexington to Clay City using these routes?
We can unfold the map to make the possibilities clearer:

```
  I-64
   /   \
  US-60 KY-89
       /   \
      KY-15 KY-402

  I-64
   /   \
  US-60 KY-89
       /   \
      KY-15 KY-402
```

The choices are independent.
6.3: Trees for counting

- We can unfold the map to make the possibilities clearer:

```
 I-64
  `- US-60
      `- KY-89
          `- KY-402
          `- KY-15

 I-64
  `- KY-89
      `- KY-402
      `- KY-15
```

- This is a decision tree. Note how the decision to be made after I-64 is the same as the decision to be made after US-60. The first choice does not affect the second choice. The choices are **independent**.
A standard Kentucky license plate has three digits followed by three letters. Assuming all choices of digits and letters were allowed, how many license plates are possible?
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\[(10) \cdot (10) \cdot (10) \cdot (26) \cdot (26) \cdot (26) = 17,576,000\]
6.3: License to count

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- How many cars are in Kentucky?
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\[(10) \cdot (10) \cdot (10) \cdot (26) \cdot (26) \cdot (26) = 17,576,000\]

How many cars are in Kentucky?

4 million people, about 4 million vehicles, 2 million of which probably have standard plates
6.3: Calorie counting

- If a restaurant offers 5 appetizers, 10 entrees, and 6 desserts, how many full course meals are possible?

\[(5)(10)(6) = 360 \text{ vs. } (5)(11)(6) = 330 \text{ vs. } (5)(10)(7) = 350\]

- If two people go to the restaurant and refuse to order the same appetizer, entree, or dessert, how many possible orders can the two people make?

\[(5)(10)(6) \text{ for the first, but one appetizer, one entree, and one dessert is now forbidden} \]

\[(5)(10)(6) - (4)(9)(5) = 54000.\]
If a restaurant offers 5 appetizers, 10 entrees, and 6 desserts, how many full course meals are possible?

If that restaurant wanted the greatest increase in the number of possibilities, should it add 1 appetizer, 1 entre, or 1 dessert?
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How many ways to arrange the letters RGB using three at a time?

Six possibilities for first (H,O,R,S,E,Y) and five for second (the remaining five)
6.3: Rearranging letters

- How many ways to arrange the letters RGB using three at a time?

- RGB, RBG, GRB, GBR, BRG, BGR
6.3: Rearranging letters

- How many ways to arrange the letters RGB using three at a time?
- RGB, RBG, GRB, GBR, BRG, BGR

Three possibilities for first (R, G, or B), and for each first letter, two choices for second (the other two), and only one choice for third letter (the only remaining one)
6.3: Rearranging letters

- How many ways to arrange the letters **RGB** using three at a time?
  - **RGB, RBG, GRB, GBR, BRG, BGR**
  - Three possibilities for first (**R, G, or B**), and for each first letter, two choices for second (the other two), and only one choice for third letter (the only remaining one).

- How many ways to arrange **HORSEY** using two at a time?
  - **HO, HR, HS, HE, HY, OH, OR, OS, OE, OY, RH, RO, RS, RE, RY, SH, SO, SR, SE, SY, EH, EO, ER, ES, EY, YH, YO, YR, YS, YE**
  - Six possibilities for first (**H, O, R, S, E, Y**) and five for second (the remaining five).
6.3: Rearranging letters

- How many ways to arrange the letters RGB using three at a time?
  - RGB, RBG, GRB, GBR, BRG, BGR

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  - RH, RO, RS, RE, RY,
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  - YH, YO, YR, YS, YE
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