MA162: Finite mathematics

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April 18, 2012

Schedule:
- HW 7A, 7B due Fri, April 20, 2012
- HW 7C due Fri, April 27, 2012
- Final exam, Wed May 2, 2012 from 8:30pm to 10:30pm

Today we will cover 7.3: Rules of probability
Final Exam Breakdown

- Chapter 7: Probability
  - Counting based probability
  - Counting based probability
  - Empirical probability
  - Conditional probability

- Cumulative
  - Ch 2: Setting up and reading the answer from a linear system
  - Ch 3: Graphically solving a 2 variable LPP
  - Ch 4: Setting up a multi-var LPP
  - Ch 4: Reading and interpreting answer form a multi-var LPP
7.2: Just count for probability

- If everything in the sample space is equally likely, then:

\[ P = \frac{\# \text{ good}}{\text{Total } \#} \]

- Probability of \( \begin{array}{c} \text{.} \\ \text{.} \end{array} \) or \( \begin{array}{c} \text{.} \\ \text{.} \end{array} \) when you roll a white and a blue die?
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- The second row and the fifth column work: \( P = \frac{6+6-1}{(6)(6)} = \frac{11}{36} \)
7.2: Crazy counting

- Suppose a deck of cards has four suits (♡, ♦, ♣, ♠) and 6 numbers (A,2,3,4,5,6)
- What is the probability of getting at least 2 aces out of 3 cards?
- Two ways to get at least 2 aces: exactly 2 or exactly 3.
Suppose a deck of cards has four suits (♥, ♦, ♣,♠) and 6 numbers (A,2,3,4,5,6).

What is the probability of getting at least 2 aces out of 3 cards?

Two ways to get at least 2 aces: exactly 2 or exactly 3.

\[
P(\text{exactly 2}) = \frac{C(4, 2)C(20, 1)}{C(24, 3)} = \frac{(4)(3)(20)}{(2)(1)(1)} = \frac{30}{506}
\]

\[
P(\text{exactly 3}) = \frac{C(4, 3)C(24, 3)}{C(24, 3)} = \frac{(4)(3)(2)}{(3)(2)(1)} = \frac{1}{506}
\]

\[
P(\text{at least 2}) = P(\text{exactly 2}) + P(\text{exactly 3}) = \frac{30}{506} + \frac{1}{506} = \frac{31}{506}
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P(\text{exactly 2}) = \frac{C(4, 2)C(20, 1)}{C(24, 3)} = \frac{(4)(3)(20)}{(2)(1)(1)} \cdot \frac{(24)(23)(22)}{(3)(2)(1)} = \frac{30}{506}
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P(\text{at least 2}) = \frac{C(4, 2)C(20, 1) + C(4, 3)}{C(24, 3)} = \frac{30}{506} + \frac{1}{506} = \frac{31}{506}
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If $P(E) = 40\%$, $P(F) = 55\%$, and $P(E \cup F) = 85\%$, then what is $P(E \cap F)$?
7.3: What if things are not equally likely?

- If \( P(E) = 40\% \), \( P(F) = 55\% \), and \( P(E \cup F) = 85\% \), then what is \( P(E \cap F) \)?

- Pretend there are 100 things total. 40 in \( E \), 55 in \( F \), 85 in \( E \cup F \).
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- So $P(E \cap F) = 10\%$, since $40\% + 55\%$ is 10\% too big.
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- What is $P(E - F)$? We definitely don’t subtract 55% from 40%. 

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\[
P(E - F) = P(E) - P(E \cap F) = 40\% - 10\% = 30\%
\]
If \( Pr(E) \) is the probability that \( E \) happens, then \( 1 - Pr(E) \) is the probability that it does not
7.3: The shortcuts

- If \( \Pr(E) \) is the probability that \( E \) happens, then \( 1 - \Pr(E) \) is the probability that it does not happen.

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- $\Pr(E) = \Pr(E \cap F) + \Pr(E - F)$
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- $Pr(E) = Pr(E \cap F) + Pr(E - F)$

- Every counting problem formula you can imagine has a probability counterpart.
What is the probability of rolling at least one six if you try 3 times?
7.3: Not not, who’s there?

- What is the probability of rolling at least one six if you try 3 times?
- You could count the number of ways, I got 91 out of 216 ways.
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You can use the first shortcut: \textbf{At least once} = \textbf{Not never}
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Never means every time it did NOT happen
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$1 - \frac{1}{6}$ chance of not happening once
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- \(1 - \frac{1}{6}\) chance of not happening once
- \((1 - \frac{1}{6})^3\) chance of it not-happening three times in a row
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\( (1 – \frac{1}{6})^3 \) chance of it not-happening three times in a row

1 – \( (1 – \frac{1}{6})^3 \) chance of THAT not happening

\[
\frac{91}{216} = 1 – \left(1 – \frac{1}{6}\right)^3
\]
Suppose 40% of people like the letter E, 55% of people like the letter F, but 15% of people don’t like either letter.
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What is the probability a random citizen likes at least one of the letters?
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100% − 15% = 85% don’t like none (so like one)
7.3: Old exam examples

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What is the probability a random citizen likes E but not F?
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  \[ 40\% - 10\% = 30\% \]
The noble knight, Vey, asked his knightly buddies how many horses they had.
7.3: Sir Vey and his noble steed

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- What is the probability a random knight had 3 or fewer steeds?

90% = 10% had 4 or more
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  90% – 40% = 50% had 3 or fewer, but not fewer.
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