Chapter 3
Notes for Instructors

Content
The third chapter of Long and DeTemple discusses place value and algorithms for whole number arithmetic. I tested students on the material from the first four sections. I spent one class discussing material from 3.5. I think this is a fun topic. This discussion was pretty relaxed so I only included one question from 3.5 on the exam. I did not have time to cover the material in section 3.6. Moreover, it is difficult to cover the calculator sections because we do not require that the students use a specific type of calculator.

Notes and Suggestions
Notes on Section 3.1: Numeration Systems Past and Present

- In section 3.1, Long and DeTemple introduce some of the numeration systems that have been used throughout history. At first glance, it may appear that this section is extraneous and unnecessary, but I have found that this section really does provide a foundation for the positional system. It also introduces the concept of borrowing in subtraction and making exchanges with addition.

- I tested students over the ideas from section 3.1, but I did not require that they memorize the notation from section 3.1 because I do not think the systems themselves are the point of section 3.1.

- Manipulatives: Several types of manipulatives are discussed at the end of section 3.1. My personal favorites are the classroom abacus and the base ten blocks. We do not have a classroom abacus, though you can improvise with some colored block and a sheet of paper. We do have a set of base ten blocks. The also mention Unifix™ cubes. I do not believe that we have any of these, but we do have multilink cubes which could be used in a similar manner.

- An Activity Note: I have provided you with a worksheet that I had my students do in class. I did not lecture on section 3.1. I found that it was better to have them work through the ideas of this section in small groups. Of course, I was willing to answer questions, but they should be able to figure out a lot of the ideas from the reading. I brought the Unifix block to class for the last question on the worksheet. I doubt the designer of these blocks intended them for this purpose, but they did help the students.

Notes on Section 3.2: Nondecimal Positional Systems

- In section 3.2, students study the concept of place value in nondecimal system. Normally students find the nondecimal positional systems frustrating. It is not uncommon to hear comments along the lines of “I will never teach this stuff to my students” and “Who would ever use this stuff?” With regards to the latter comment, you can
tell them that computer scientists use the base two and base sixteen systems. With regards to the former comment, you can tell them that you are trying to help them understand how their students will feel when they begin to learn about the decimal system. Remember, that the decimal system may be familiar to us, but it will not be familiar to their students. Moreover, we want these future teachers to know more about place value than their students.

- **An Activity Note with Manipulatives:** They do not introduce place value cards here, but I like these because they can be easily adjusted for problems in different bases and they are easy to make. All you need is a piece of paper and some poker chips. Long and DeTemple discuss place value cards in sections 3.3 and 3.4, but it does not hurt to mention them earlier. Place value cards can be very helpful when converting between different bases. For example, suppose you want to find the base six representation for fifty-four. Start with a pile of fifty-four poker chips. Place these in the ones place of a base six place value card. When we are finished, no place should have more than five poker chips. Remove a group of thirty-six chips from the ones place and exchange it for one chip in the thirty-sixes place. This will leave eighteen chips in the ones place and one chip in the thirty-sixes place. Now remove a group of six chips from the ones place and replace it with one chip in the sixes place. Do this two more times. You will then have a place value card with one chip in the thirty-sixes place and three chips in the sixes place. Therefore the base six representation of fifty-four is $130_{\text{six}}$.

**Notes on Sections 3.3 and 3.4:** *Algorithms for Adding and Subtracting Whole Numbers* and *Algorithms for Multiplication and Division of Whole Numbers*

- I spent several days on the material from sections 3.3 and 3.4. Students should be able to use manipulatives to justify algorithms for addition, subtraction, multiplication, and division of whole numbers. Moreover, they need to understand that the traditional algorithms for computations with whole number are not the only valid algorithms.

- I am particularly fond of problem 14 in section 3.3. I was surprised at the number of future teachers who would have told the hypothetical student in this problem that he or she had incorrect reasoning. Many of the future teachers believed that the hypothetical student had only happened on the correct answer by luck.

- **An Activity Note with Manipulatives:** As I mentioned before, I like to use place value cards because they are easy to make and I find that students understand them. Certainly you can use other manipulatives such as base ten blocks (assuming that you are, in fact, working in base ten) in a similar manner. I am providing you with four examples of arithmetic problems which I have solved with place value cards to show you how you can use these cards with students.