Exam #1 Review

Chapter 1

1. Strategies for solving problems (pp. 15–18).

2. Application of these strategies for a complete solution with explanation of the Five Planes Problem, and indeed of the general \( n \) Planes Problem (determining the maximum number of regions of space that can be created by cutting it with \( n \) planes), and how this problem relates to the lower-dimensional problem of determining the maximum number of regions of the plane that can be created by cutting it with \( n \) lines, which we also completely solved.

3. In the process, we learned the formula for \( 1 + 2 + 3 + \cdots + n \) and an explanation of why the formula is correct.

4. Problems 5, 6, 8, 17, 22, 24.

Chapter 2

1. How we can use physical methods of measuring the circumference of a circular object.

2. How we can approximate the circumference of a circle with the perimeters of regular \( n \)-gons, and why this shows (via similar triangles) that the ratio of circumference to diameter is the same for any size circle, thus defining \( \pi \).

3. How we can use physical methods of measuring the area of a circular object.

4. How we can approximate the area of a circle using grids, or inscribing or circumscribing squares.

5. Why slicing a circle into wedges strongly suggests that \( A = \frac{1}{2} r C \), and how we can then use this to get a formula for the area of a circle.

6. The formulas for the volumes of prisms, pyramids, cylinders, and cones.

7. How we can use physical methods of measuring the volume of a spherical object.

8. Knowing that the volume of a pyramid depends only on the area of a base and the height from that base, why the three pyramids that dissect the triangular prism (pp. 74–78) each have equal volume.

9. Why slicing a solid ball into pyramidal wedges strongly suggests that \( V = \frac{1}{3} r A \).
10. How we can find a formula for the volume of a sphere using Cavalieri’s principle, and from this and the previous equation, how we can get a formula for the surface area of a sphere.

11. How area changes if we scale every length of a plane figure by a factor of $k$. How volume and surface area change if we scale every length of a three-dimensional figure by a factor of $k$.

12. Problems 1, 3, 6, 8, 9, 14, 15, 16, 17, 22, 36, 37, 46, 48.