MA/CS415 Homework #5
Due Wednesday, September 29

1. Continue reading and studying pages 30–51.

2. Page 33, #1.

3. Page 37, #6, 10.

4. Page 42, #5.

5. Let $G$ be a simple graph with vertices $\{1, \ldots, n\}$. Suppose every edge $e$ of $G$ is assigned a nonnegative cost $c(e)$. Define the cost of any walk to be the sum of the costs of its edges. Define the matrix $C$ with entries

$$C_{ij} = \begin{cases} 
0 & \text{if } i = j, \\
c(e) & \text{if } i \neq j, \ e = \{i, j\} \in E(G), \\
+\infty & \text{if } i \neq j, \ \{i, j\} \not\in E(G).
\end{cases}$$

Let $C^{(k)}$ be the matrix obtained by raising $C$ to the $k$ power using weird matrix multiplication. Prove that the $ij$ entry of $C^{(k)}$ is the cost of a minimum cost walk from $i$ to $j$ using at most $k$ edges. Suggestion: Refer to the previous homework, and also take into account the effect of the zeroes down the main diagonal.