MA515 Homework #6
Due Monday, October 17


2. Consider the linear programs $(P)$ and $(P(u))$:

\[
\begin{align*}
\text{max } c^T x & \quad \text{max } c^T x \\
\text{s.t. } Ax &= b & \text{s.t. } Ax = b + u \\
\quad x &\geq O & \quad x \geq O \\
\end{align*}
\]

$(P)$ \hspace{1cm} $(P(u))$

Assume that $(P)$ has an optimal objective function value $z^*$. Suppose that there exists a vector $y^*$ and a positive real number $\varepsilon$ such that the optimal objective function value $z^*(u)$ of $(P(u))$ equals $z^* + u^T y^*$ whenever $\|u\| < \varepsilon$. Prove that $y^*$ is an optimal solution to the dual of $(P)$. 