1. Let us assume we have an LP of the form

\[
\begin{align*}
\text{max } z &= c^T x \\
\text{s.t. } Ax &= b \\
x &\geq O
\end{align*}
\]

(a) Let \( \bar{x} \) be a feasible point. Prove that \( \bar{x} \) is a basic feasible solution if and only if it is a vertex (using our earlier definition of vertex involving \( N(\bar{x}) \)).

(b) Assume that we have a basic feasible solution \( \bar{x} \) associated with some basis \( B \), and that we also have some basic direction \( \bar{w} \) associated with \( B \) and nonbasic \( s \in N \). For convenience, let us also assume that \( \bar{x}_j > 0 \) for each \( j \in B \) and that \( \bar{w} \) is not nonnegative. So when we consider the ray \( \bar{x} + t\bar{w}, t \geq 0 \), we will discover some leaving variable \( x_r, r \in B \). Prove that \( B' = (B \cup \{s\}) \setminus \{r\} \) is a basis; i.e., prove that the columns of \( A_{B'} \) are linearly independent.

2. Exercise 8.3.


4. Exercise 10.2.