Two Polyhedral Construction Problems
Carl Lee

Two problems I am presently interested in both involve developing methods of construction of polyhedra. At the moment I have more questions than answers.

1. \textbf{f-vectors of Regular Triangulations.} For a vector \( a = (a_0, \ldots, a_m) \) define \( g(a) = (a_0 - a_m, a_1 - a_{m-1}, \ldots, a_k - a_{m-k}) \), where \( k = \lfloor (m - 1)/2 \rfloor \). Suppose \( h = (h_0, \ldots, h_d) \) is the \( h \)-vector of a regular triangulation. Then it is known that \( g(h, 0, \ldots, 0) \) is an \( M \)-vector, regardless of the number of 0’s appended to \( h \). This is proved as a consequence of the \( g \)-Theorem for simplicial polytopes and repeated application of the co-wedge operation, and also reflects certain properties of the weight algebra. Laura Schmidt and I are investigating to what extent these conditions may be sufficient. The first step involves attempting to construct a simplicial complex with the desired \( h \)-vector; the second, its realization as a regular triangulation. We have been focusing on extracting the simplicial complex as a partial shelling of the \( g \)-Theorem polytopes using various shelling orders of them.

2. \textbf{cd-indices of 4-polytopes.} Bisztriczky introduced ordinary polytopes as nonsimplicial analogs of cyclic polytopes. Dinh’s geometric construction can be viewed as a certain “sewing” procedure. Matt Menzel and I are considering various modifications to the sewing procedure to see what changes in the \( cd \)-index can result, and which \( cd \)-indices can be constructed in this manner.