Worksheet # 5: Trigonometric Integrals

1. What are the three Pythagorean trigonometric identities?

2. What are the power reduction (half angle) formulas for $\sin^2 x$ and $\cos^2 x$?

3. Evaluate the following integrals.
   
   (a) $\int \cos^2(x) \, dx$
   
   (b) $\int \sin^3 x \, dx$
   
   (c) $\int \sin^4(x) \, dx$
   
   (d) $\int_0^{\pi/2} \sin^2 x \cos^2 x \, dx$
   
   (e) $\int \tan^2 x \, dx$
   
   (f) $\int \sin^3 x \cos^2 x \, dx$
   
   (g) $\int x^2 \sin x \, dx$
   
   (h) $\int \frac{\sin \phi}{\cos^3 \phi} \, d\phi$
   
   (i) $\int \tan^3 x \sec^3 x \, dx$
   
   (j) $\int \sin 8x \cos 5x \, dx$  \hspace{1cm} (Hint: Use sum and difference formulas)
   
   (k) $\int \frac{1 - \tan^2 x}{\sec^2 x} \, dx$
   
   (l) $\int t \sec^2(t^2) \tan^4(t^2) \, dt$
   
   (m) $\int_0^\pi \cot^3 x \, dx$
   
   (n) $\int \sec^3 x \, dx$  \hspace{1cm} (Hint: Start out using integration by parts with $u = \sec x$)

4. Prove that for $m, n$ integers
   
   $$\frac{1}{\pi} \int_0^{2\pi} \cos(mx) \cos(nx) = \begin{cases} 0 & m \neq n \\ 1 & m = n \end{cases}.$$

5. Evaluate $\int \sin x \cos x \, dx$ by four methods:

   (a) the substitution $u = \cos x$
   
   (b) the substitution $u = \sin x$
   
   (c) the identity $\sin 2x = 2 \sin x \cos x$
   
   (d) integration by parts