MA109, Activity 1: Basic Equations (Section 1.1, pp. 74-80)  

Today's Goal: Equations are the basic mathematical tool for solving real-world problems. We introduce them and we learn how to solve some special classes.

Assignments: Homework (Sec. 1.1): # 1, 4, 7, 15, 18, 20, 23, 28, 33, 40, 49, 55, 62, 88 (pp. 80-81).

An equation is a statement that two mathematical expressions are equal. For instance:

\[ 4^3 - 2 \cdot 2^2 = 32. \]

Most equations that we study in Algebra contain variables. For example,

\[ x^2 + 2x = 32. \]

Given an equation in the variable \(x\), the goal is to find the values of \(x\) that make the equation true. These values are called the solutions or roots of the equation, and the process of finding the solutions is called solving the equation.

**Example 1:** Using the previous terminology, we say that \(x = 4\) is a solution (or root) of the equation:

\[ x^2 - 2x = 32. \]

**Example 2:** Determine whether the given value of \(x\) is a solution of the equation:

\[ 4 - 2 \cdot (3 + x) = 12 - (16 + x). \]

(a) \(x = 2\)

\[ \text{Left} = 1 - [2 - (3 - 2)] = 1 - [2 - 1] = 1 - 1 = 0 \]

\[ \text{Right} = 4 - 2 - (6 + 2) = 8 - 8 = 0 \]

Since Left = Right, \(x = 2\) is a solution.

(b) \(x = 4\)

\[ \text{Left} = 1 - [2 - (3 - 4)] = 1 - [2 - (-1)] = 1 - 3 = -2 \]

\[ \text{Right} = 4 - (6 + 4) = 16 - 10 = 6 \]

Left ≠ Right, so \(x = 4\) is not a solution.

**Properties of Equations:**

1. \(A + B \rightarrow A = B = C\)
2. \(A \cdot B \rightarrow AC = BC\) \((C \neq 0)\)

**Types of Equations:**

- **Linear Equations:**
  
  A linear equation (or first-degree equation) in one variable is an equation equivalent to one of the form:

  \[ ax + b = 0, \]

  where \(a\) and \(b\) are real numbers and \(x\) is the variable. [Divide \(a\) sides \(x \neq 0)\].
Example 3: The given equations are linear or equivalent to linear equations. Solve these equations:

\[ \begin{align*}
5t - 13 &= 12 - 5t \\
10t - 15 &= 12 \\
10t &= 25 \\
t &= \frac{25}{10} \\
t &= 2.5
\end{align*} \]

\[ \begin{align*}
(x + 3)^2 &= (x - 1)^2 - 8 \\
x^2 + 6x + 9 &= (x^2 - 2x + 1) - 8 \\
8x + 9 &= -7 \\
8x &= -16 \\
x &= -2
\end{align*} \]

\[ \begin{align*}
2y - \frac{1}{3}y - 3y &= \frac{y + 1}{4} \\
12 \cdot \frac{2}{3}y + 12 \cdot \frac{1}{2}y - 3y &= 12 \cdot \frac{y + 1}{4} \\
8y + 6(y - 3) &= 3(y + 1) \\
14y - 18 &= 3y + 3 \\
y &= 2 \frac{1}{11}
\end{align*} \]

Now we consider basic equations that can be simplified into the form \( X^n = a \) where \( n > 1 \).

These equations can be solved by taking radicals of both sides of the equation. We can also solve simple equations involving a fractional power of the variable.

Example 4: The given equations involve a power of a variable. Find all real solutions of these equations:

\[ \begin{align*}
\sqrt{y} &= 5 \\
y^{\frac{1}{3}} &= 5 \\
(y^{\frac{1}{3}})^3 &= (5)^3 \\
y &= 125
\end{align*} \]
\[ x^4 - 16 = 0 \]
\[ x^4 = 16 \]
\[ x = \pm \sqrt[4]{16} \]
\[ x = \pm 2 \]

(Alternatively, factor the polynomial \( x^4 - 16 \) above and obtain your solutions from such factorization.)

**Example 5:** The average daily food consumption \( F \) of an herbivorous mammal with body weight \( w \), where both \( F \) and \( w \) are measured in pounds, is given approximately by the equation

\[ F = 0.3w^{4/3} \]

Find the weight \( w \) of an elephant who consumes 300 lbs of food per day.

\[ 300 = 0.3w^{4/3} \]
\[ w^{4/3} = \frac{300}{0.3} = 1000 \]
\[ w = 1000^{3/4} \]

**Example 6:** Solve the given equation for the indicated variable.

- \( x^4 - 16 = 0 \)
- \( \frac{a^2 + b}{a + d} = 2 \)
- \( \frac{a^2 + a + b}{a - c} = \frac{b + c}{a - d} \)
- \( a + b = 2(c + d) \)
- \( a^2 - 2cx = 2d - b \)
- \( (a - c)x = 2d - b \)
- \[ x = \frac{2d - b}{a - c} \]

- \( \frac{a}{b} \)
- \( a\left(\frac{q + 1}{b}\right) = ab \cdot \frac{q - 1}{b} + a\cdot b + b \cdot \frac{q + 1}{b} \)
- \[ a^2 + a = a^2 - a + b(b + 1) \]
- \[ 2a = b(b + 1) \]
- \[ a = b(b + 1)/2 \]
- \( a^{-1} \)
- \( \frac{a^{-1}}{(a - 1)(a + 1)} = \frac{a^{-1}}{2} \)
- \( a^2 - (a + 1)x = -c \)
- \[ (a^2 - (a + 1))x = 1 - a \]
- \[ x = \frac{1 - a}{a^2 - a - 1} \]