MA 110 — Precalculus Fall 2014
Exam 4 17 December 2014

Name: ________________________________

Section: _____________________________

Last 4 digits of student ID #: __________

This exam has twelve multiple choice questions (five points each) and five free response questions (eight points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:
1. You must give your final answers in the multiple choice answer box on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the multiple choice answer box.

On the free response problems:
1. Clearly indicate your answer and the reasoning used to arrive at that answer (unsupported answers may not receive credit).
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

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Record the correct answer to the following problem on the front page of this exam.

(1) Using complex numbers, solve the quadratic equation exactly and express answer in standard complex form.

$$2x^2 + 6x + 6 = 0.$$ 

A) \(\frac{-3 + \sqrt{3}i}{2}\)

B) The equation has no solution

C) \(-6 \pm \sqrt{21}\)

D) \(-3 \pm \frac{\sqrt{3}i}{2}\)

E) \(-3 \pm \frac{\sqrt{12}i}{4}\)

(2) Find the equation of the line passing through (-1, 2) and parallel to the line passing through the points (-2, -4) and (2, 8).

A) \(y = 6x + 5\)

B) \(y = \frac{1}{2}x - 1\)

C) \(y = 3x + 5\)

D) \(y = \frac{1}{6}x + 1\)

E) \(y = 3x - 7\)

(3) Find the average rate of change of the \(f(x) = x^2 - 1\) as \(x\) changes from \(x = -2\) to \(x = 3\).

A) 1

B) -1

C) -2

D) \(\frac{3}{5}\)

E) 5
(4) Which one of the following is a factor of \( p(x) = x^8 - 100x - 56\)?

A) \( x - 1 \)
B) \( x - 3 \)
C) \( x - 2 \)
D) \( x + 1 \)
E) \( x + 2 \)

\[
p(1) = 1 - 100 - 56 \neq 0 \]
\[
p(2) = 256 - 200 - 56 = 0 \]

2 is root
So \((x - 2)\) is factor

(5) Determine the end behavior of

\[
\frac{3x^4 - 2x + 1}{5x^4 - 5x^3 + 1}
\]

\[\frac{d}{dx}y = y^3\]

as \(x \to -\infty\).

A) \( y \to \infty \)
B) \( y \to -\infty \)
C) \( y \to 0 \)
D) \( y \to 1 \)
E) \( y \to \frac{3}{5} \)

(6) Simplify

A) \( 2xy \)
B) \( -2xy \)
C) \( \frac{x^2}{y} \)
D) \( 2x - y \)
E) \( x^2 - y \)

\[e^{2\ln(x) - \ln(y)} = e^{\ln x^2 - \ln y} = \frac{x^2}{y}\]
Record the correct answer to the following problem on the front page of this exam.

(7) Solve for $x$ exactly.

$$\ln(x) + \ln(x - 1) = \ln(2)$$

\[ \ln(x(x - 1)) = \ln(2) \]
\[ x^2 - x = 2 \]
\[ x^2 - x - 2 = 0 \]
\[ (x - 2)(x + 1) \]

A) 2 only.
B) $\sqrt{e^2 + 1}$
C) $\frac{1}{2}$
D) 2 and $-1$.
E) The equation has no solution.

\[ \frac{\ln(-1)}{2} \text{ undefined} \]

(8) Solve the absolute value inequality.

$$|2x - 4| \leq 8$$

A) $(-\infty, 4) \cup [12, \infty)$
B) $[-2, 6]$  
C) $(-\infty, -2] \cup [12, \infty)$
D) The inequality has no solution.
E) $(-2, 6)$

(9) Find the solution to

$$\frac{x - 1}{x + 2} \geq 0$$

A) $[1, \infty)$
B) $(-\infty, -2) \cup [1, \infty)$
C) $(-\infty, -2] \cup [1, \infty)$
D) $(-2, 1)$
E) $[-2, 1]$
(10) Find \( \tan \left( \frac{-\pi}{6} \right) \) exactly.

A) \( \frac{\sqrt{3}}{3} \)

B) \( \frac{-\sqrt{2}}{2} \)

C) \( \frac{\sqrt{3}}{3} \)

D) \( \frac{-\sqrt{3}}{3} \)

E) \( -1 \)

(11) Determine which one of the following statements is false.

\( \neg \ A \) \( \cot \left( \frac{21\pi}{4} \right) = -1 \)

\( \top \) B) \( \sin(x) = \cos \left( x - \frac{\pi}{2} \right) \) for all values of \( x \).

\( \top \) C) \( \sin(x) > 0 \) for \( 0 < x < \pi \).

\( \top \) D) \( \cos(x) = \cos \left( x - 8\pi \right) \) for all values of \( x \).

\( \top \) E) \( \sec \left( \frac{\pi}{2} \right) \) is undefined.

(12) Find \( \sin(t) \) if \( \cos(t) = \frac{\sqrt{2}}{3} \) and \( t \) is in the fourth quadrant.

A) \( \frac{\sqrt{7}}{3} \)

B) \( \frac{-1}{3} \)

C) \( -\frac{\sqrt{2}}{3} \)

D) \( \frac{\sqrt{11}}{3} \)

E) \( \frac{-\sqrt{7}}{3} \)

\[ \sin(t) = -\sqrt{1 - \left( \frac{\sqrt{2}}{3} \right)^2} = -\sqrt{1 - \frac{2}{9}} = -\sqrt{\frac{7}{9}} = -\frac{\sqrt{7}}{3} \]
Free Response Questions: Show your work!

(13) Given

\[ f(x) = -4 \sin \left(3x - \frac{\pi}{2}\right) \]

1. Find the amplitude, period, and phase shift for \( f(x) \).

\[ \text{amp} = 4 \]
\[ \text{period} = \frac{2\pi}{3} \]

2. Label the five tick marks indicated on the \( x \)- and \( y \)-axes on the graph below.
(14) Solve the triangle $ABC$ shown below. Round answers to two decimal places. The angle is in degrees so make sure your calculator is in degree mode! (The triangle may not be to scale.)

\[ a = 12.72 \quad \angle A = 60^\circ \]

\[ b = 8.3 \quad \angle B = 42.1^\circ \]

\[ c = 12.1 \]

\[ C = 77.88^\circ \]

\[ \cos(C) = \frac{c^2 - a^2 - b^2}{-2ab} = \frac{12.1^2 - (12.777)^2 - (8.3)^2}{-2(12.777)(8.3)} = 0.209 \]

\[ C = \cos^{-1}(0.209) = 77.88^\circ \]

\[ B = 180 - A - C = 42.12^\circ \]
Free Response Questions: Show your work!

(15) Find the time it takes for an investment of $2000 to grow to $3,500 if it is put into an account with a 5.1% interest rate compounded continuously. Solve algebraically and show all work. Your answer should be exact.

\[ 3500 = 2000e^{0.051t} \]

\[ \frac{35}{20} = e^{0.051t} \]

\[ \ln(1.75) = 0.051t \]

\[ \frac{\ln(1.75)}{0.051} = t \approx 10.97 \text{ years} \]
Free Response Questions: Show your work!

(16) Find the center and the radius of the circle given by

\[ x^2 - 2x + y^2 + 4y - 5 = 0. \]

\[ \begin{align*}
(x - 1)^2 &+ (y + 2)^2 = 5 + 1 + 4 \\
\left(\frac{x}{2}\right)^2 &+ \left(\frac{y}{2}\right)^2 = 2^2 = 4 \\
\end{align*} \]

Center \( (1, -2) \)

Radius = \sqrt{10}
Free Response Questions: Show your work!

(17) (a) Find the difference quotient \( \frac{f(x + h) - f(x)}{h} \) for

\[ f(x) = 2x^2 - 5. \]

\[
\frac{2(x + h)^2 - 5 - (2x^2 - 5)}{h} = \frac{2(x^2 + 2xh + h^2) - 5 - 2x^2 + 5}{h}
\]

\[ = \frac{2x^2 + 4xh + h^2 - 2x^2 + 5}{h} = \frac{4xh + h^2}{h} = 4x + h. \]

(b) Use your answer from part (a) to find the average rate of change of \( f(x) \) as \( x \) changes from \( x = -3 \) to \( x = -1 \). Indicate the values of \( h \) and \( x \).

\[
\begin{align*}
    x &= -3, \\
    h &= -1 - (-3) = 2, \\
    f(x) &= f(-3) + 2 = 1 - 10 = -9.
\end{align*}
\]
Some Useful Formulas

\[ B(t) = P (1 + r)^t \]
\[ B(t) = P \left( 1 + \frac{r}{n} \right)^{nt} \]
\[ P(t) = P_0 e^{rt} \]
\[ Q(t) = Q_0 e^{-rt} \]

\[ a^2 = b^2 + c^2 - 2bc \cos(A) \]
\[ b^2 = a^2 + c^2 - 2ac \cos(B) \]
\[ c^2 = a^2 + b^2 - 2ab \cos(C) \]

\[ \frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c} \]