This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

**On the multiple choice problems:**

1. You must give your final answers in the multiple choice answer box on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the multiple choice answer box.

**On the free response problems:**

1. Clearly indicate your answer and the reasoning used to arrive at that answer (unsupported answers may not receive credit).
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

### Multiple Choice Answers

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Record the correct answer to the following problems on the front page of this exam.

1. Let \( f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 - 10x \). Which of the following statements must be true?

   (A) \( f \) has critical points at \(-2, 5\) and \(\frac{3}{2}\).
   (B) \( f \) has critical points at \(-2\) and \(5\), and an inflection point at \(\frac{3}{2}\).
   (C) \( f \) has critical points at \(-2\) and \(5\), but has no inflection points.
   (D) \( f \) has an inflection point at \(\frac{3}{2}\), but has no critical points.
   (E) \( f \) has no critical points and no inflection points.

2. The linear approximation to \( f(x) = \sqrt{x^2 + 3} \) at \( x = 1 \) is

   (A) \( L(x) = x + 2 \)
   (B) \( L(x) = \frac{1}{2}x + \frac{1}{2} \)
   (C) \( L(x) = \frac{1}{2}x + \frac{3}{2} \)
   (D) \( L(x) = \frac{1}{4}x + \frac{1}{2} \)
   (E) \( L(x) = x + \frac{1}{2} \)

3. \( f(x) \) is continuous on \([0, 2]\) and differentiable on \((0, 2)\). Assume that \( f(0) = 1 \) and \( f(2) = 5 \). Which of the following statements must be true?

   (A) \( f(x) \) is increasing on \((0, 2)\).
   (B) There is a point \( c \) in \((0, 2)\) such that \( x = c \) is a critical point.
   (C) There is a point \( c \) in \((0, 2)\) such that \( x = c \) is a local max or a local min.
   (D) There is a point \( c \) in \((0, 2)\) such that \( x = c \) is an inflection point.
   (E) There is a point \( c \) in \((0, 2)\) such that \( f'(c) = 2 \).
4. Find \( \lim_{x \to +\infty} \frac{(4x - 2)(x + 3)(x - 4)}{(2x + 3)(x + 1)(x - 5)}. \)

(A) 0  
(B) 1  
(C) 2  
(D) 4  
(E) +\infty

5. The value of \( \lim_{x \to 0^+} x^{\frac{1}{2}} \ln x \) is

(A) \( -\infty \)  
(B) 0  
(C) \( \frac{1}{2} \)  
(D) 1  
(E) \( \infty \)
6. Find \( \lim_{t \to 0} \frac{e^{t^2} - 1}{t^2} \).

(A) 0
(B) 1
(C) 2
(D) 3
(E) +\infty

7. Let \( f(x) = x^2 e^x - 1 \). We want to use Newton’s method to solve \( f(x) = 0 \). Let \( x_0 = 1 \). Find \( x_1 \) rounded to the nearest 3 digits after the decimal point.

(A) 0.143
(B) 0.332
(C) 0.511
(D) 0.789
(E) 0.923
8. Assume that the functions $f(x)$ and $f'(x)$ are each differentiable everywhere. If $a$ and $b$ are two critical points of $f(x)$ (where $a < b$) and $f(x)$ is not constant on $(a, b)$, which of the following statements must be true?

(A) $f(x)$ has at least one additional critical point in $(a, b)$.
(B) $f(x)$ has at least one inflection point in $(a, b)$.
(C) The absolute maximum of $f(x)$ on $[a, b]$ occurs at either $a$ or $b$.
(D) $f(x)$ has either a local max or a local min in $(a, b)$.
(E) $f(x)$ has no additional critical points in $(a, b)$.

9. Given that $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$, find $\sum_{k=10}^{15} (2k + 3)$.

(A) 148
(B) 152
(C) 158
(D) 162
(E) 168

10. The antiderivative $\int x^2 + \sin(2x - 7) \, dx$ is equal to

(A) $x^3 - \cos(2x - 7) + C$
(B) $\frac{1}{3}x^3 - 2\cos(2x - 7) + C$
(C) $\frac{1}{3}x^3 + 2\cos(2x - 7) + C$
(D) $\frac{1}{3}x^3 + \frac{1}{2}\cos(2x - 7) + C$
(E) $\frac{1}{3}x^3 - \frac{1}{2}\cos(2x - 7) + C$
11. In parts (a) and (b), let \( f(x) = \tan x \).

   (a) Find \( L(x) \) where \( L(x) \) is the linear approximation of \( f(x) = \tan x \) at \( x = 0 \).

   (b) Compute \( \lim_{x \to 0} \frac{f(x) - L(x)}{x} \).
12. A manufacturer wants to design an open box that has a square base, no top, and a surface area of 108 square inches.

(a) Sketch the box and label its dimensions.

(b) Write the equation stating that the surface area of the box is 108 square inches.

(c) What dimensions will produce a box with the maximum volume?
13. Let \( f(x) = -x^4 + 4x^3 + 4353. \)

(a) Find the intervals over which \( f(x) \) is increasing, and find the intervals over which \( f(x) \) is decreasing. Find all local minima and all local maxima.

(b) Find the intervals over which \( f(x) \) is concave up, and find the intervals over which \( f(x) \) is concave down. Find all inflection points.
14. The acceleration of a particle moving along the $x$-axis is given by the equation $a(t) = -3t^2 + 2t$. At $t = 0$, the particle is at $x = 1$ with a velocity of 5 m/s.

(a) Find the velocity function $v(t)$ of this particle and use it to find $v(2)$.

(b) Find the position function $s(t)$ of this particle and use it to find $s(2)$. 
Free Response Questions: Show your work!

15. Consider the function \( f(x) = x^2 \). We are interested in the area \( A \) that is under the graph of \( f(x) \) and above the interval \([0, 4]\).

(a) Divide the interval \([0, 4]\) into \( n \) intervals of equal length and write the expression for \( R_n \), the sum that represents the right-endpoint approximation to the area \( A \).

(b) Use the fact that \( \sum_{j=1}^{N} j^2 = \frac{N(N+1)(2N+1)}{6} \) to find a closed form expression for \( R_n \).

(c) Take the limit of \( R_n \) as \( n \) tends to infinity to find an exact value for the area \( A \).