Answer all of the questions 1 - 7 and two of the questions 8 - 10. Please indicate which problem is not to be graded by crossing through its number in the table below.

Additional sheets are available if necessary. No books or notes may be used. Please, turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities. Please:

1. clearly indicate your answer and the reasoning used to arrive at that answer (unsupported answers may not receive credit),

2. give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Name: ________________________________

Section: _________

Last four digits of student identification number: __________

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<th>Question</th>
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(1) Compute the following limits:

(a) \( \lim_{x \to 1} \frac{\ln x}{\sin(\pi x)} \)

(b) \( \lim_{x \to \infty} x \sin \left( \frac{\pi}{x} \right) \)

(c) \( \lim_{t \to \infty} (\sqrt{t^2 + 9} - t) \)

(a) \( \lim_{x \to 1} \frac{\ln x}{\sin(\pi x)} = \)

(b) \( \lim_{x \to \infty} x \sin \left( \frac{\pi}{x} \right) = \)

(c) \( \lim_{t \to \infty} (\sqrt{t^2 + 9} - t) = \)
(2) (a) Let \( f(x) = x^9 g(2x) \). If \( f'(1) = 1 \) and \( g(2) = -1 \), determine \( g'(2) \).

(b) If \( h(x) = \cos(\ln x - e^{x^2}) \), compute \( h'(x) \).

(a) \( g'(2) = \) ____________________________.

(b) \( h'(x) = \) ____________________________.
(3) Compute the following integrals:

(a) \[ \int \left( \sin x - \frac{1}{x^2 + 1} + 2e^x \right) \, dx \]

(b) \[ \int t \sqrt{t^2 + 4} \, dt \]

(c) \[ \int_{1}^{x} \frac{1 - t \cos t}{t} \, dt \]
(4) Consider the curve described by the equation $e^{xy} = x - y$.

(a) Find the slope of the tangent line to the curve at the point $(0, -1)$.

(b) Determine the equation of the tangent line to the curve at $(0, -1)$ in the form $y = mx + b$.

(a) The slope is _________________.

(b) The equation of the tangent line is _________________.

5
(5) (a) Let \( F(x) = \int_1^x \frac{t}{(t + 3)^2} \, dt \), where \( x > -3 \) Determine the interval(s) on which \( F(x) \) is increasing.

(b) Let \( G(x) = \int_x^3 \left( \frac{1 - t}{t} \right)^2 \, dt \), where \( x > 0 \). Compute \( G''(x) \).

(a) \( F(x) \) is increasing on ________________________.

(b) \( G''(x) = \) ________________________.
Consider the function $f(x) = \frac{1}{x^2}$.

(a) Compute the Riemann sum for $f(x)$ on the interval $[1, 5]$ with $n = 4$ subintervals and by taking the left endpoints as your sample points. (Give your answer as a rational number.)

(b) Determine whether $f(x)$ is increasing or decreasing on $[1, 5]$.

(c) Without computing the integral $\int_1^5 f(x) \, dx$, determine whether the Riemann sum of Part (a) overestimates or underestimates this integral.

(a) The Riemann sum is ___________________________.

(b) $f(x)$ is ___________________________.

(c) The Riemann sum ___________________________ the integral.
Let \( a \) and \( b \) be positive numbers whose product is 8. Find the minimum value of \( a^2 + 2b \). Determine the values of \( a \) and \( b \) for which the minimum is attained. (As usual, justify your answers.)

The minimum value of \( a^2 + 2b \) is \( \___ \). It is attained if \( a = \___ \) and \( b = \___ \).
Work two of the following three problems. Indicate the problem that is not to be graded by crossing through its number on the front of the exam.

(8) (a) State both parts of the fundamental theorem of calculus. Use complete sentences and be sure to include all assumptions.

(b) Compute the derivative of \( F(x) = \int_0^x \sin(\sqrt{t}) \, dt \) at \( x = \frac{\pi^2}{4} \).

(c) Find the derivative of \( G(x) = \int_1^{x^2} (\ln t)^3 \, dt \).

(b) \( F'(x) = \quad \) 

(c) \( G'(x) = \quad \)
(9) (a) What does it mean for a function $f(x)$ to be continuous at a point $a$? Use complete sentences.

(b) Consider the piecewise defined function

$$f(x) = \begin{cases} 
2 & \text{if } 0 \leq x \leq 1, \\
ax + 3 & \text{if } 1 < x \leq 2, \\
x^2 - 2x + b & \text{if } 2 \leq x \leq 3,
\end{cases}$$

where $a$ and $b$ are constants. Find the values of $a$ and $b$ for which $f(x)$ is continuous on $[0, 3]$.

(b) $a = \underline{\phantom{0000}}$ and $b = \underline{\phantom{0000}}$. 
A particle is moving along the $x$ axis with velocity $v(t) = 3t^2 + 3t - 6$ at time $t$ (measured in meters per second).

(a) Find the acceleration $a(t)$ of the particle at time $t = 2$.

(b) Is the particle speeding up or slowing down at time $t = 2$? Justify your answer.

(c) Find all times $t$ at which the particle changes direction. Again, justify your answer.

(d) Find the total distance traveled by the particle during the first 3 seconds.

(a) $a(2) =$ ____________________________ .

(b) the particle is _______________________ .

(c) the particle changes direction at ________ .

(d) the total distance traveled is ___________ .
Extra Credit Problem:
Check the correct answers below. For each correct answer you earn 2 points, and for each incorrect answer 1 point will be subtracted. Therefore, it might be wise to skip a question rather than risking losing a point. However, your final score on this problem will not be negative! You need not justify your answer.

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☐ ☐ If \( f(2) < 0 \) and \( f(5) > 0 \), then there exists a number \( c \) between 2 and 5 such that \( f(c) = 0 \).

☐ ☐ If \( f'(c) = 0 \), then \( f(x) \) has a local maximum or minimum at \( x = c \).

☐ ☐ If \( f \) is differentiable function and \( f(-2) = f(2) \), then there exists a number \( c \) such that \( |c| < 2 \) and \( f'(c) = 0 \).

☐ ☐ \( \int \ln x \, dx = x \ln x - x + C \).

☐ ☐ \( \int_{-3}^{3} (ax^2 + c) \, dx = 2 \int_{0}^{3} (ax^2 + c) \, dx \) for every choice of \( a \) and \( c \).