Worksheet # 3: Limits: A Numerical and Graphical Approach

1. Comprehension check:
   (a) In words, describe what \( \lim_{x \to a} f(x) = L \) means.
   (b) In words, what does \( \lim_{x \to a} f(x) = \infty \) mean?
   (c) Suppose \( \lim_{x \to 1} f(x) = 2 \). Does \( f(1) = 2 \)?
   (d) Suppose \( f(1) = 2 \). Does \( \lim_{x \to 1} f(x) = 2 \)?

2. Compute the value of the following functions near the given \( x \)-value. Use this information to guess the value of the limit of the function (if it exist) as \( x \) approaches the given value.
   (a) \( f(x) = (x - 2)^3 - 1, \ x = 1 \)
   (b) \( f(x) = \frac{4x^2 - 9}{2x - 3}, \ x = \frac{3}{2} \)
   (c) \( f(x) = \frac{x}{1}, \ x = 0 \)
   (d) \( f(x) = 2^{x-1} + 1, \ x = 1 \)
   (e) \( f(x) = \frac{x^2 - 3x + 2}{x^2 + x - 6}, \ x = 2 \)

3. Let \( f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ x - 1 & \text{if } 0 < x \text{ and } x \neq 2 \\ -3 & \text{if } x = 2 \end{cases} \)
   (a) Sketch the graph of \( f \).
   (b) Compute the following:
      i. \( \lim_{x \to 0^-} f(x) \)
      ii. \( \lim_{x \to 0^+} f(x) \)
      iii. \( \lim_{x \to 0} f(x) \)
      iv. \( f(0) \)
      v. \( \lim_{x \to 2^-} f(x) \)
      vi. \( \lim_{x \to 2^+} f(x) \)
      vii. \( \lim_{x \to 2} f(x) \)
      viii. \( f(2) \)

4. In the following, sketch the functions and use the sketch to compute the limit.
   (a) \( \lim_{x \to 3} \pi \)
   (b) \( \lim_{x \to \pi} x \)
   (c) \( \lim_{x \to |x|} \)
   (d) \( \lim_{x \to 3} 2^x \)

5. Compute the following limits or explain why they fail to exist:
   (a) \( \lim_{x \to -3^+} \frac{x + 2}{x + 3} \)
   (b) \( \lim_{x \to -3^-} \frac{x + 2}{x + 3} \)
   (c) \( \lim_{x \to -3} \frac{x + 2}{x + 3} \)
   (d) \( \lim_{x \to 0^-} \frac{1}{x^3} \)
6. In the theory of relativity, the mass of a particle with velocity $v$ is:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where $m_0$ is the mass of the particle at rest and $c$ is the speed of light. What happens as $v \to c$?

7. Let $f(x) = \begin{cases} 
2x + 2 & \text{if } x > -2 \\
3 & \text{if } x = -2 \\
kx & \text{if } x < -2 
\end{cases}$.

Find $k$ and $a$ so that \( \lim_{x \to -2} f(x) = f(-2) \).