1. (5 points.) The purpose of this exercise is to prove the power law for derivatives of $x^n$ using the Principle of Induction and the product rule for derivatives. The Principle of Induction states the following. Suppose $P_n$ is a set of statements depending on $n = 1, 2, 3, \ldots$. We would like to verify that they are true. However, it is impossible to verify infinitely many statements one at a time! Instead, suppose we can verify 1) the statement $P_1$ is true, and, 2) suppose that if we assume $P_n$ is true, then we can use this information to verify that $P_{n+1}$ is also true. In this case, the Principle of Induction states that all the statements $P_n$ are true!

(a) Let $f_n(x) = x^n$, for $n = 1, 2, 3, \ldots$. We want to use the Principle of Induction and the product rule to show that

$$f'_n(x) = nx^{n-1}, \quad n = 1, 2, 3, \ldots.$$ 

Formulate a statement $P_n$ that will imply the power law stated above.

(b) Verify the truth of the first statement $P_1$.

(c) Assume that statement $P_n$ is true. Use the truth of $P_n$ and the product rule to show that $P_{n+1}$ is true.

(d) Conclude that $P_n$ holds for any $n = 1, 2, 3, \ldots$ by the Principle of Induction.

2. (5 points.) Find the coordinates of a point $Q$ on the graph of the function $f(x) = x^2$ so that the secant line through the points $(-1, 1)$ and $Q$ has the same slope as the tangent to the graph of the function $f$ at the point $(2, 4)$. 