(1) The form of the partial fraction decomposition of the rational function

\[ f(x) = \frac{3x + 2}{(x+1)^2(x^2 + 3)} \]

with the parameters \( A, B, C, D, E \) being constants to be determined, is:

A) \( \frac{A}{(x+1)^2} + \frac{Bx+C}{x^2 + 3} \)
B) \( \frac{A}{x+1} + \frac{Bx+C}{(x+1)^2} + \frac{D}{x^2 + 3} \)
C) \( \frac{A}{x+1} + \frac{Bx+C}{(x+1)^2} + \frac{Dx+E}{x^2 + 3} \)
D) \( \frac{A}{(x+1)^2} + \frac{B}{x^2 + 3} \)
E) none of the above

(2) Which of the following statements is false?

A) \( \int_1^\infty \frac{dx}{x^2} \) converges
B) \( \int_0^1 \frac{dx}{x^{4/3}} \) diverges
C) \( \int_1^2 \frac{dx}{(x-1)^2} \) diverges
D) \( \int_2^4 \frac{dx}{x-2} \) converges
E) \( \int_2^\infty \frac{dx}{\sqrt{x^2 - 1}} \) diverges

\[ \int_2^\infty \frac{dx}{x^2 - 1} \]

\[ = \lim_{w \to 0^+} \frac{1}{\ln(w)} = -\infty \]
(3) Which of the integrals below represents the length of the curve $y = \tan x$ from $x = 0$ to $x = \pi/4$?

A) $\int_{0}^{\pi/4} \sqrt{1 + \tan^2 x} \, dx$

B) $\int_{0}^{\pi/4} \sqrt{1 + \tan^2 x \sec^2 x} \, dx$

C) $\int_{0}^{\pi/4} \sqrt{1 + \sec^4 x} \, dx$

D) $2\pi \int_{0}^{\pi/4} \tan x \sqrt{1 + \tan^2 x} \, dx$

E) $\int_{0}^{\pi/4} x \tan x \, dx$

$$y' = \sec^2 (x)$$
$$\left( y' \right)^2 = \left( \sec^2 (x) \right)^2$$
$$L = \int_{0}^{\pi/4} \sqrt{1 + \left( \sec^2 x \right)^2} \, dx$$

(4) A triangular laminar (thin plate) of uniform mass density has its vertices at the points $A = (0, 3)$, $B = (0,0)$, and $C = (1,0)$ in the $x$-$y$ plane. Where is the center of mass of the laminar located?

A) $(0,1)$

B) $\left( \frac{1}{3}, 1 \right)$

C) $\left( \frac{1}{3}, \frac{3}{2} \right)$

D) $\left( \frac{2}{3}, 1 \right)$

E) $\left( \frac{1}{3}, \frac{3}{2} \right)$

$$\begin{align*}
M_x &= \rho \int_{0}^{3} y \left( \frac{x}{3} + 1 \right) \, dy \\
&= \rho \int_{0}^{3} \left( -\frac{y^2}{3} + \frac{y}{2} \right) \, dy = \rho \left( \frac{1}{3} + \frac{3}{2} \right) = \rho \left( \frac{3}{2} \right) \\
M_y &= \rho \int_{0}^{1} x \left( -3x + 3 \right) \, dx = \rho \int_{0}^{1} \left( -9x^2 + 3x \right) \, dx = \rho \left( -3 + \frac{3}{2} \right) = \rho \left( \frac{3}{2} \right)
\end{align*}$$

$$\text{cm} = \left( \frac{1}{3}, 1 \right)$$
(5) Which of the following differential equations is NOT separable?

A) \( xy' + y = y^2 \)
B) \( (1 + x^2) y' = x^3 y \)
C) \( x(y^2 - 1) + y(x^2 - 1)y' = 0 \)
D) \( y' = \sin y \)

(6) Which of the following statements is false? In what follows, \( k \) and \( b \) are given constants, and \( C \) stands for an arbitrary constant.

A) The general solution of the differential equation \( y' = k(y - b) \) is \( y = b + Ce^{kt} \).
B) The general solution of the differential equation \( y' = k(y - b) \) is \( y = b - Ce^{kt} \).
C) The general solution of the differential equation \( y' = -k(y - b) \) is \( y = b + Ce^{-kt} \).
D) If \( k > 0 \), then all solutions of \( y' = k(y - b) \) tend to \( \infty \) as \( t \to \infty \).
E) If \( k > 0 \), then all solutions of \( y' = -k(y - b) \) approach the same limit as \( t \to \infty \).
(7) Evaluate the integral
\[ \int \frac{3x}{(x-1)(x^2+2)} \, dx. \]

\[ \frac{2x}{(x-1)(x^2+2)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+2} \]

\[ 3x = A(x^2+2) + (Bx+C)(x-1) \]

\[ x=1 \quad 3 = A(3) \Rightarrow A = 1 \]

\[ x=0 \quad 0 = A(2) + C(-1) \Rightarrow C = 2 \]

\[ x=-1 \quad -3 = A(3) + (-B+2)(-2) \Rightarrow B = -1 \]

\[ \int \left( \frac{1}{x-1} + \frac{-x+2}{x^2+2} \right) \, dx = \int \frac{1}{x-1} \, dx - \int \frac{x}{x^2+2} \, dx + 2 \int \frac{dx}{x^2+2} \]

\[ u = x^2 + 2 \]
\[ \frac{du}{2} = x \, dx \]

\[ = \ln |x-1| - \frac{1}{2} \ln (x^2+2) + 2 \int \frac{dx}{x^2+2} \]

\[ \int \frac{dx}{x^2+2} = \frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{x}{\sqrt{2}} \right) \]

\[ \int \frac{dx}{x^2+2} = \frac{1}{2} \ln (x^2+2) + \sqrt{2} \tan^{-1} \left( \frac{x}{\sqrt{2}} \right) + C \]
Free Response Questions: Show your work!

(8) Compute the surface area of the surface obtained by rotating the graph of \( y = \sqrt{1 + 2x} \) about the \( x \)-axis over the interval \([0, 1]\).

\[
y' = \frac{1}{2} (1 + 2x)^{-\frac{1}{2}} (2) = (1 + 2x)^{-\frac{1}{2}}
\]

\[
(y')^2 = (1 + 2x)^{-1}
\]

\[
S = 2\pi \int_0^1 y \sqrt{1 + (y')^2} \, dx = 2\pi \int_0^1 \frac{\sqrt{1 + 2x}}{1 + 2x} \, dx
\]

\[
= 2\pi \int_0^1 \frac{\sqrt{2 + 2x}}{1 + 2x} \, dx
\]

\[
= 2\pi \int_0^1 \frac{\sqrt{2 + 2x}}{2} \, dx = 2\pi \int_0^1 (2 + 2x)^{\frac{1}{2}} \, dx
\]

\[
n = 2 + 2x
\]

\[
dx = 2 \, dx
\]

\[
= 2\pi \left( \frac{2}{3} \right) \left( 2 + 2x \right)^{\frac{3}{2}} \bigg|_0^1
\]

\[
= 2\pi \left( 4^{\frac{3}{2}} - 2^{\frac{3}{2}} \right) = \frac{2\pi}{3} \left( 5, 17157 \right)
\]

\[
\approx 10.8313
\]
Free Response Questions: Show your work!

(9)  The following table gives the measured values of a force function \( f(x) \), where \( x \) is in meters and \( f(x) \) in newtons.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>10.0</td>
<td>9.5</td>
<td>9.3</td>
<td>9.1</td>
<td>9.2</td>
</tr>
</tbody>
</table>

\[ \Delta x = 2 \] meters

(a) Use Simpson’s Rule to estimate the work done by the force \( f \) in moving an object from \( x = 0 \) to \( x = 8 \) meters.

\[
W \approx \frac{\Delta x}{3} \left( f(0) + 4f(2) + 2f(4) + 4f(6) + f(8) \right)
\]

\[
= \frac{2}{3} \left( 10.0 + 4(9.5) + 2(9.3) + 4(9.1) + 9.2 \right)
\]

\[
= \frac{2}{3}(112.2) = 74.8 \text{ Newtons}
\]

(b) It is known that the force function \( f(x) \) satisfies the inequality \(|f^{(4)}(x)| \leq 2\) on the interval \([0,8]\). Let \( S_N \) be the \( N \)th approximation to \( \int_0^8 f(x)dx \) by Simpson’s rule. Use the given inequality on \(|f^{(4)}(x)|\) to find the smallest \( N \) that guarantees \( \text{Error}(S_N) \leq 10^{-1} \). (Hint: Use the error bound for \( S_N \) given on the last page of the exam.)

\[
\text{Error}_N \leq \frac{K_4 (b-a)^5}{180 N^4} \leq \frac{1 f^{(4)}(c) |(8-0)^5}{180 (N^4)}
\]

\[
\text{Error}_N \leq \frac{2 (8)^5}{180N^4} \leq 10^{-1}
\]

\[
\frac{2 (8^5)}{180 (10^{-1})} \leq N^4
\]

\[
N \geq \sqrt[4]{\frac{2 (8^5)}{180 (10)}} \approx 7.767
\]

\[ \Rightarrow \text{smallest } N = 8 \]
(10) Let $T_n(x)$ ($n = 0, 1, 2, \cdots$) be the $n$th Taylor polynomial for $f(x) = e^x$ centered at $a = 0$.

(a) Find the Taylor polynomial $T_n(x)$.

\[
\begin{align*}
    f(0) &= e^0 = 1 \\
    f'(0) &= e^0 = 1 \\
    f''(0) &= e^0 = 1 \\
    &\vdots \\
    f^{(n)}(0) &= e^0 = 1 \\
\end{align*}
\]

(b) Find a value of $n$ for which

\[
|e^x - T_n(x)| \leq 10^{-2}
\]
on the interval $[0, 1]$. (Hint: Use the error bound given on the last page of the exam.)

\[
10^{-2} \leq \frac{f^{(k+1)}(\xi) |x-0|^n}{(n+1)!}
\]

\[f^{(k+1)}(1) = e^1 = e\]

\[
10^{-2} \leq \frac{e^n}{(n+1)!}
\]

\[
(n+1)! \leq e^{(100)} = 271.83
\]

\[\text{when } n = 6 \quad \text{answer}
\]

\[
(n+1)! = 7! = 5040
\]

\[\text{when } n = 5 \quad (n+1)! = 6! = 720 \quad \text{not big enough}
\]
Free Response Questions: Show your work!

(11) Solve the initial value problem

\[ \frac{dx}{dt} = x^2(1-t^2), \quad x(1) = 1. \]

\[ \frac{dy}{x^2} = (1-t^2) \, dt \]

\[ \int \frac{dy}{x^2} = \int (1-t^2) \, dt \]

\[ \frac{-1}{x} = t - \frac{t^3}{3} + C \]

\[ x = 1, \quad t = 1 \]

\[ \frac{-1}{1} = 1 - \frac{1}{3} + C \]

\[ -\frac{5}{3} = C \]

\[ \frac{-1}{x} = t - \frac{t^3}{3} - \frac{5}{3} \]

\[ x = \frac{-1}{t - \frac{t^3}{3} - \frac{5}{3}} \]