- Each question is followed by space to write your answer. Write your solutions neatly in the space below the question.
- Clearly indicate your answer and the reasoning used to arrive at that answer. Unsup-ported answers may not receive credit.
- Unless a problem specifically asks for an approximation, you must give exact answers to receive credit.
- You may use a calculator, but not one which has symbolic manipulation capabilities.
- Turn off your cell phones, and any other electronic devices which can send and receive wireless signals. You may not wear ear-plugs during the exam.
- No books or notes may be used.

Name: ______________________________

Section: __________

Last four digits of student identification number: _________

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$\sin^2 A + \cos^2 A = 1$
$1 + \cot^2 A = \csc^2 A$
$\tan^2 A + 1 = \sec^2 A$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\sin(2A) = 2 \sin A \cos A$
$\cos(2A) = \cos^2 A - \sin^2 A$
1. (a) Evaluate the series \( \sum_{n=2}^{\infty} \frac{4}{n^2 - 1} \)

(b) Determine whether or not the series \( \sum_{k=0}^{\infty} \frac{\cos k}{e^k} \) converges. Make sure to state the test(s) that you use, and verify that their assumptions are satisfied.
2. Let \( R \) be the (unbounded) region which lies below the curve \( y = x^{-1.5} \), above the \( x \)-axis, and to the right of the line \( x = 1 \). Hint: For the problems below, think of \( R \) as being bounded on the right by the line \( x = a \) for some large \( a \), and then let \( a \to \infty \).

(a) Consider the solid obtained by revolving \( R \) about the \( x \)-axis. Determine whether or not this solid has finite volume. If it does, compute it. Make sure to clearly indicate the relevant integral.

(b) Consider the solid obtained by revolving \( R \) about the \( y \)-axis. Determine whether or not this solid has finite volume. If it does, compute it. Make sure to clearly indicate the relevant integral.
3. An oddly-shaped well is 50 feet deep, and water (which weighs 62 pounds per cubic foot) fills the bottom 40 feet. Let $A = A(x)$ be the cross-sectional area (in square feet) of the well with respect to the height $x$ (in feet) from the bottom of the well.

(a) The work required to empty the well is given by an integral of the form

$$\int_a^b c(r - x) A(x) \, dx .$$

Give the values of the constants $a$, $b$, $c$ and $r$.

(b) Use Simpson’s rule and the measurements below to estimate the work needed to pump all of the water from the well.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
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<tr>
<td>$A$</td>
<td>30</td>
<td>25</td>
<td>30</td>
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</table>
4. Give the Taylor polynomial of degree 2, centered at 0, for the function \( f(x) = (2 - x)^\pi \).

5. Set up an integral for evaluating the arclength of the cycloid \( x = t - \sin t, \ y = 1 - \cos t \) for \( 0 \leq t \leq 2\pi \). Simplify the integrand using basic trigonometric identities. Do not evaluate this integral.
6. Consider the polar curves \( r = \sin(2\theta) \) and \( r = \cos \theta \), which are pictured below.

(a) Determine the Cartesian coordinates \((x, y)\) of the point of intersection which is strictly in the first quadrant, i.e. \( x, y > 0 \).

(b) Set up an integral, or integrals, for computing the area of the region in the first quadrant between the bolded portion of the two curves. \textit{Do not evaluate the integral(s)}. 

\[ 
\]
7. The Bessel function $J_0$ is given by the power series $J_0(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)^2} \left( \frac{x}{2} \right)^{2n}$.

(a) Determine the interval of convergence for this series.

(b) Letting $S_N$ denote the $N^{th}$ partial sum, determine the smallest integer $N$ for which it is guaranteed that $|J_0(1/2) - S_N(1/2)| < 10^{-16}$.
8. Consider the seasonal-growth model \( \frac{dP}{dt} = kP \cos(rt) \), \( P(0) = P_0 \), where \( k, r \) and \( P_0 \) are positive constants.

(a) Give a formula for \( P \) in terms of \( t, k, r \) and \( P_0 \).

(b) Taking the values \( r = 2, k = 1 \) and \( P_0 = 100 \), estimate \( P(\pi/2) \) using Euler’s method, with step-size \( h = \pi/4 \). You may give your answer as a decimal approximation (providing at least three digits beyond the decimal).
9. Evaluate each of the definite or indefinite integrals below:

(a) \( \int x \arctan x \, dx \)

(b) \( \int_{0}^{\pi/4} \tan^3 x \sec^3 x \, dx \)