Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (a) is correct, you must write

\[ \text{a} \text{ b} \text{ c} \text{ d} \text{ e} \]

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

**GOOD LUCK!**

1. [ ] a [ ] b [ ] c [ ] d [ ] e  
2. [ ] a [ ] b [ ] c [ ] d [ ] e  
3. [ ] a [ ] b [ ] c [ ] d [ ] e  
4. [ ] a [ ] b [ ] c [ ] d [ ] e  
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19. [ ] a [ ] b [ ] c [ ] d [ ] e  
20. [ ] a [ ] b [ ] c [ ] d [ ] e

For grading use:

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1
Please make sure to list the correct section number on the front page of your exam.
In case you forgot your section number, consult the following table.

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Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam and in the corresponding questions that follow.

1. Find the derivative, \( f'(x) \), if \( f(x) = \sqrt{4x^3 + 5x^2 + 6x + 2} \).

   \[
   f'(x) = \frac{1}{2} \left( 4x^3 + 5x^2 + 6x + 2 \right)^{\frac{-1}{2}} \cdot \left( 12x^2 + 10x + 6 \right)
   \]

   Possibilities:
   
   (a) \( \frac{1}{2} \left( 4x^3 + 5x^2 + 6x + 2 \right)^{\frac{-1}{2}} \cdot \left( 12x^2 + 10x + 6 \right) \)
   
   (b) \( 12x^2 + 10x + 6 \)
   
   (c) \( 4x^3 + 5x^2 + 6x + 2 \)
   
   (d) \( \sqrt{12x^2 + 10x + 6} \)
   
   (e) \( \frac{1}{2} \left( 4x^3 + 5x^2 + 6x + 2 \right)^{\frac{-1}{2}} \cdot \left( 12x^2 + 10x + 6 \right) \)

2. Find the derivative, \( f'(x) \), if \( f(x) = e^{2x^3 + 6x^2 + 7x} \).

   \[
   f'(x) = \frac{d}{dx} e^{2x^3 + 6x^2 + 7x} = e^{2x^3 + 6x^2 + 7x} \cdot (6x^2 + 12x + 7)
   \]

   Possibilities:

   (a) \( e^{2x^3 + 6x^2 + 7x} \cdot (6x^2 + 12x + 7) \)
   
   (b) \( 2x^3 + 6x^2 + 7x \)
   
   (c) \( e^{6x^2 + 12x + 7} \)
   
   (d) \( 6x^2 + 12x + 7 \)
   
   (e) \( 2x^3 + 6x^2 + 7x \)

3. For the function \( f(x) = 2x^3 + 4x^2 + 3x + 1 \), find the equation of the tangent line to graph of \( f \) at \( x = 3 \).

   Possibilities:

   (a) \( y = x^3 + 17 \)
   
   (b) \( y = 81x - 143 \)
   
   (c) \( y = 100x - 219 \)
   
   (d) \( y = 100 \)
   
   (e) \( y = 81x + 100 \)

   \[
   y - \text{value of point} = f(3) = 2 \cdot 3^3 + 4 \cdot 3^2 + 3 \cdot 3 + 1 = 2 \cdot 27 + 4 \cdot 9 + 9 + 1 = 100
   \]

   \[
   f'(x) = 6x^2 + 8x + 3
   \]

   \[
   f'(3) = 6 \cdot 3^2 + 8 \cdot 3 + 3 = 6 \cdot 9 + 24 + 3 = 81
   \]

   \[
   m = f'(3) = 81
   \]

   \[
   y - y_1 = m(x - x_1)
   \]

   \[
   y - 100 = 81(x - 3)
   \]
4. Suppose \( F(x) = \ln(g(x)) \). If \( g(2) = 7, \ g'(2) = 11, \) and \( g''(2) = 3 \), then find \( F'(2) \).

**Possibilities:**

(a) \( 7/11 \)
(b) \( \ln(7)/11 \)
(c) \( 11/7 \)
(d) \( \ln(3) \)
(e) \( 7/\ln(11) \)

\[
F'(x) = \frac{1}{g(x)} \cdot g'(x)
\]

**Rule for \( \ln \) derivative of \( g \)**

\[
F'(2) = \frac{1}{g(2)} \cdot g'(2) = \frac{1}{7} \cdot (11) = \frac{11}{7}
\]

5. Suppose \( F(x) = g(x) \cdot h(x + 2) \). If \( g(0) = 6, \ g'(0) = 3, \ h(0) = 8, \ h'(0) = 7, \ h(2) = 4, \) and \( h'(2) = 9 \), find \( F'(0) \).

**Product Rule**

\[
F'(x) = g'(x) \cdot h(x + 2) + h'(x + 2) \cdot g(x)
\]

**Possibilities:**

(a) 130
(b) 120
(c) 74
(d) 37
(e) 66

\[
F'(0) = g(0) \cdot h'(0 + 2) + h(0 + 2) \cdot g'(0)
\]

\[
= 6 \cdot 9 + 4 \cdot 3
\]

\[
= 54 + 12 = 66
\]

6. Suppose \( F(x) = (g(x))^5 + 7 \). If \( g(2) = 9, \ g'(2) = 13, \) and \( g''(2) = 3 \), then find \( F'(2) \).

**Possibilities:**

(a) \( (5)(9^4) + 7 \)
(b) 3
(c) \( 9^5 + 7 \)
(d) \( 13^5 + 7 \)
(e) \( (5)(9^4)(13) \)

\[
F'(x) = 5 (g(x))^4 \cdot g'(x) + 0
\]

**Power Rule**

**Inside derivative of 7**

\[
F'(2) = 5 (g(2))^4 \cdot g'(2) + 0
\]

\[
= 5 \cdot 9^4 \cdot 13
\]
7. If \( f(x) = \frac{8}{x+5} \) then choose the simplified form of \( \frac{f(x+h)-f(x)}{h} \):

\[
f(x+h) = \frac{8}{x+h+5}, \quad \text{So}
\]

Possibilities:

(a) \( \frac{16x+80+8h}{(x+h+5)(x+5)(2x+h)} \)

(b) \( \frac{h^2+10hx+25h-8}{(x+5)^2} \)

(c) \( -\frac{8}{(x+h+5)^2} \)

(d) \( -\frac{8}{(x+h+5)(x+5)} \)

(e) \( \frac{8}{(x+h+5)(x+5)} \)

\[
f(x+h) - f(x) = \frac{8}{x+h+5} - \frac{8}{x+5}
\]

\[
= \frac{8(x+5)}{(x+h+5)(x+5)} - \frac{8(x+h+5)}{(x+h+5)(x+5)}
\]

\[
= \frac{8x+40 - (8x+8h+40)}{(x+h+5)(x+5)}
\]

\[
= \frac{8h}{(x+h+5)(x+5)}
\]

\[
\therefore \quad \frac{1}{h} = \frac{-8h}{(x+h+5)(x+5)}
\]

8. Find the derivative, \( f'(x) \), if \( f(x) = (6 + 9x)e^{2+9x} \).

Product Rule

Possibilities:

(a) \( \frac{9}{2+9x} \)

(b) \( (63 + 81x)e^{2+9x} \)

(c) \( (81)e^9 \)

(d) \( (9)e^9 \)

(e) \( (9)e^{2+9x} \)

\[
f'(x) = (6 + 9x) \cdot e^{2+9x} \cdot 9 + e^{2+9x} \cdot 9
\]

\[
= 9(6 + 9x) + e^{2+9x}
\]

\[
= e^{2+9x} \left( 9(6 + 9x) + 9 \right)
\]

\[
= e^{2+9x} (54 + 81x + 9)
\]

\[
= e^{2+9x} (81x + 63)
\]

9. Find the derivative, \( f'(x) \), of \( f(x) = \frac{1}{x^8} = x^{-8} \)

Possibilities:

(a) \( -8x^{-9} \)

(b) \( -8x^{-7} \)

(c) \( 8x^7 \)

(d) \( 1/(8x^7) \)

(e) \( 1/(8x^9) \)

\[
f'(x) = -8x^{-9}
\]

power rule.
10. For the function \( f(x) = \begin{cases} x^2 - 4 & x < 10 \\ \sqrt{x + 7} & 10 \leq x < 20 \\ x^3 - 6 & 20 \leq x \end{cases} \), find the equation of the tangent line to the graph of \( f \) at \( x = 19 \).

**Possibilities:**

(a) \( y = 357x - 2818 \)

(b) \( y = 1083x - 13724 \)

(c) \( y = \sqrt{26x - \frac{415}{52}} \sqrt{26} \)

(d) \( y = 38x - 365 \)

(e) \( y = \frac{1}{52} \sqrt{26x} + \frac{33}{52} \sqrt{26} \)

For these \( x \), \( f(x) = (x + 7)^{\frac{1}{2}} \)

So \( f'(x) = \frac{1}{2} (x + 7)^{-\frac{1}{2}} (1) = \frac{1}{2\sqrt{x+7}} \)

and \( m = f'(19) = \frac{1}{\sqrt{19+7}} = \frac{1}{\sqrt{26}} \cdot \frac{\sqrt{26}}{52} = \frac{1}{52} \)

\( y \)-value is \( f(19) = \sqrt{19+7} = \sqrt{26} \)

\( y - \sqrt{26} = \frac{1}{52} (x - 19) \Rightarrow y = \frac{1}{52} x - \frac{19}{52} \sqrt{26} + \frac{\sqrt{26}}{52} \)

11. Find the derivative, \( f'(x) \), if \( f(x) = (7 + 9x) \ln(6 + 7x) \).

**Possibilities:**

(a) \( \frac{16}{6 + 7x} \)

(b) \( \frac{1}{x} \)

(c) \( \frac{9}{6 + 7x} \)

(d) \( (9 \ln(6 + 7x)) + \frac{49 + 63x}{6 + 7x} \)

(e) \( 9 + \frac{7}{6 + 7x} \)

**Product Rule:**

\[
\begin{align*}
\frac{d}{dx} \left( (7 + 9x) \ln(6 + 7x) \right) & = \ln(6 + 7x) \cdot (7 + 9x) \cdot \frac{d}{dx} (6 + 7x) + (7 + 9x) \cdot \frac{d}{dx} \ln(6 + 7x) \\
& = \left( 7 + 9x \right) \cdot \frac{1}{6 + 7x} + 9 \ln(6 + 7x)
\end{align*}
\]

12. For the function \( f(x) = \ln(6x^2 + 8x + 9) \), find the equation of the tangent line to graph of \( f \) at \( x = 0 \).

**Possibilities:**

(a) \( y = 3x + \ln(9) \)

(b) \( y = \ln(9) x + \frac{8}{9} \)

(c) \( y = \ln(9) \)

(d) \( y = \frac{8}{9} x + \ln(9) \)

(e) \( y = x^3 + 17 \)

\( f'(x) = \frac{1}{6x^2 + 8x + 9} \cdot 12x + 8 \)

\( m = f'(0) = \frac{1}{0 + 0 + 9} \cdot (0 + 8) = \frac{8}{9} \)

\( y \)-value \( = f(0) = \ln(0 + 0 + 9) = \ln 9 \)

\( y - \ln 9 = \frac{8}{9} (x - 0) \Rightarrow y - \ln 9 = \frac{8}{9} x \Rightarrow y = \frac{8}{9} x + \ln 9 \)
13. If \( f(x) = 9x^4 + 7x \) then find the second derivative \( f''(x) \):

Possibilities:
(a) \( 36x^3 + 7 \)
(b) \( 36x^3 + 54x^2 + 36x + 16 \)
(c) \( 108x^2 \)
(d) \( 144x^4 \)
(e) \( 108x^2 + 18 \)

\[
\begin{align*}
  f'(x) &= 36x^3 + 7 \\
  f''(x) &= 36 \cdot 3x^2 + 0 \\
          &= 108x^2
\end{align*}
\]

14. If \( f(x) = (12x + 32)^{23} \) then \( f''(x) = ? \)

Possibilities:
(a) \( 23(22)(12x + 32)^{21}(12)^2 \)
(b) \( 23^2(12)^{23}(12x + 32) \)
(c) \( 23(12x + 32)^{22} \)
(d) \( 0 \)
(e) \( 23(22)12^{21} \)

\[
\begin{align*}
  f'(x) &= 23(12x + 32)^{22} \cdot 12 \\
        &= 23 \cdot 12 \cdot (12x + 32)^{22} \\
  f''(x) &= 23 \cdot 12 \cdot 2(12x + 32)^{21} \cdot 12 \\
          &= 23 \cdot 22 \cdot 12 \cdot 12 \cdot (12x + 32)^{21}
\end{align*}
\]

15. The function \( f(x) \) is increasing on \((-\infty, 1] \cup [9, \infty)\) and decreasing on \([1, 9]\). The values \( f(1) = 11 \) and \( f(9) = 3 \) are known. Which of the following is possible?

Possibilities:
(a) \( f(11) = 2 \) \( f(6) \) is \(< 11 \)
(b) \( f(-1) = 12 \)
(c) \( f(5) = 12 \) \( f(3) \) is \(< 11 \)
(d) \( f'(5) = 7 \) \( f'(5) \) is \( \text{negative} \)
(e) \( f'(5) = -7 \)

\( \text{Only possibility} \)
16. If an amount of \( x \) dollars is invested at 2\% interest compounded continuously, and at the end of 5 years the value of the investment is $3000, find \( x \).

Possibilities:

(a) $2714.51
(b) $3315.51
(c) $300
(d) $2000
(e) $588.11

\[ P = P_0 \ e^{rt} \]

\[ 3000 = x \ e^{.02(5)} \]

\[ x = \frac{3000}{\ e^{.02(5)}} = \frac{3000}{\ e^{.1}} \approx 2714.512 \]

17. The numbers of a bacteria in a culture doubles every 13 hours. How many hours will it take before 9 times the original number of bacteria is present?

Possibilities:

(a) \( \frac{13}{9} \)
(b) \( \frac{117}{2} \)
(c) \( \frac{13}{2} \)
(d) \( 13 \ln (9)/\ln (2) \)
(e) \( 13 \ln (2)/\ln (9) \)

\[ P = P_0 \ e^{rt} \]

Suppose \( P_0 = 1 \). Then \( 2 = 1 \ e^{r(13)} \)

\[ \Rightarrow \ e^{13r} = 2 \Rightarrow \ln e^{13r} = \ln 2 \]

\[ \Rightarrow 13r = \ln 2 \Rightarrow r = \frac{\ln 2}{13} \]

Then

\[ 9 = 1 \ e^{\frac{\ln 2}{13} \cdot t} \Rightarrow \ e^{\frac{\ln 2}{13} \cdot t} = 9 \]

\[ \Rightarrow \ln e^{\frac{\ln 2}{13} \cdot t} = \ln 9 \Rightarrow \frac{\ln 2}{13} \cdot t = \ln 9 \]

\[ \Rightarrow t = \frac{\ln 9}{\frac{\ln 2}{13}} = \frac{13 \cdot \ln 9}{\ln 2} \]
18. The graph of \( y = f(x) \) is shown below. The minimum value of \( f(x) \) on the interval \([-5, 3]\) occurs at which \( x \)?

**Possibilities:**
(a) \(-2\)
(b) \(2\)
(c) \(-4\)
(d) \(0\)
(e) \(-3\)

19. Find the minimum of \( g(t) = -(t + 2)^2 + 7 \) on the interval \([-3, 0]\)

**Possibilities:**
(a) \(7\)
(b) \(6\)
(c) \(-2\)
(d) \(3\)
(e) \(0\)

\[
\begin{align*}
g'(t) &= -2(t+2)(1) + 0 = -2t-4 \\
g'(t) &= 0 \text{ when } -2t-4 = 0 \Rightarrow -2t = 4 \Rightarrow t = -2, \text{ which is in } [-3, 0]. \text{ Check endpoints and critical numbers.} \\
g(-3) &= -(-3+2)^2 + 7 = -(1)^2 + 7 = -1 + 7 = 6 \\
g(0) &= -(0+2)^2 + 7 = -4 + 7 = 3 \text{ \textbf{SMALLEST}} \\
g(-2) &= -(2+2)^2 + 7 = -0^2 + 7 = 7
\end{align*}
\]

20. Find the minimum of \( g(t) = -2t^3 - 3t^2 + 36t - 2 \) on the interval \([1, 4]\)

**Possibilities:**
(a) \(42\)
(b) \(29\)
(c) \(-34\)
(d) \(-83\)
(e) \(0\)

\[
\begin{align*}
g'(t) &= -6t^2 - 6t + 36 \\
&= -6(t^2 + t - 6) = -6(t + 3)(t - 2) \\
g'(t) &= 0 \text{ when } t = -3 \text{ (not in } [1, 4]) \text{ and when } t = 2, \text{ check endpoints and critical numbers:} \\
g(1) &= -2\cdot1^3 - 3\cdot1^2 + 36\cdot1 - 2 = -2 - 3 + 36 - 2 = 29 \\
g(4) &= -2\cdot4^3 - 3\cdot4^2 + 36\cdot4 - 2 = -128 - 48 + 144 - 2 = -34 \\
g(2) &= -2\cdot2^3 - 3\cdot2^2 + 36\cdot2 - 2 = -16 - 12 + 72 - 2 = 42
\end{align*}
\]