Answer all of the following questions. Use the backs of the question papers for scratch paper. No books or notes may be used. You may use a calculator. You may not use a calculator that has symbolic manipulation capabilities. When answering these questions, please be sure to:

- check answers when possible,
- clearly indicate your answer and the reasoning used to arrive at that answer (unsupported answers may receive NO credit).

<table>
<thead>
<tr>
<th>QUESTION</th>
<th>SCORE</th>
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<td>Bonus.</td>
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Please make sure to list the correct section number on the front page of your exam. In case you forgot your section number, consult the following table:

<table>
<thead>
<tr>
<th>Sections #</th>
<th>Lecturer</th>
<th>Time/Location</th>
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<tbody>
<tr>
<td>001-004</td>
<td>Kate Ponto</td>
<td>MWF 10:00 am - 10:50 am, CP 320</td>
</tr>
<tr>
<td>005-008</td>
<td>Alberto Corso</td>
<td>MWF 01:00 pm - 01:50 pm, CB 102</td>
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<thead>
<tr>
<th>Section #</th>
<th>Recitation Instructor</th>
<th>Time/Location</th>
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<tbody>
<tr>
<td>001</td>
<td>Dustin Hedmark</td>
<td>TR 09:30 am - 10:20 am, CB 347</td>
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<tr>
<td>002</td>
<td>Michael Gustin</td>
<td>TR 09:30 am - 10:20 am, RRH 0130</td>
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<tr>
<td>003</td>
<td>Dustin Hedmark</td>
<td>TR 11:00 am - 11:50 am, CB 347</td>
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<tr>
<td>004</td>
<td>Michael Gustin</td>
<td>TR 12:30 pm - 01:20 pm, CB 347</td>
</tr>
<tr>
<td>005</td>
<td>Liam Solus</td>
<td>TR 12:30 pm - 01:20 pm, FB 213</td>
</tr>
<tr>
<td>006</td>
<td>Liam Solus</td>
<td>TR 02:00 pm - 02:50 pm, FB 213</td>
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<tr>
<td>007</td>
<td>Joseph Lindgren</td>
<td>TR 02:00 pm - 02:50 pm, CB 245</td>
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<tr>
<td>008</td>
<td>Joseph Lindgren</td>
<td>TR 03:30 pm - 04:20 pm, FB 213</td>
</tr>
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</table>
1. The following table gives resting heart rates for various animals.

<table>
<thead>
<tr>
<th></th>
<th>mass (grams)</th>
<th>pulse rate (beats/minute)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mouse</td>
<td>25</td>
<td>668</td>
</tr>
<tr>
<td>rabbit</td>
<td>2000</td>
<td>187</td>
</tr>
<tr>
<td>large dog</td>
<td>30000</td>
<td>85</td>
</tr>
<tr>
<td>horse</td>
<td>450000</td>
<td>39</td>
</tr>
</tbody>
</table>

(a) Plot, as accurately as you can, this data on the log-log plot below.

(b) Use this graph to develop a functional relationship between mass and pulse rate.  
    (Hint: Since we are using a log-log plot we expect a power function.)
2. (a) For what choice of $c$ is the function $f(x) = \begin{cases} x^2 + c & \text{if } x < 1 \\ \frac{3}{x^2} & \text{if } x \geq 1 \end{cases}$ continuous everywhere?

(b) Use the Sandwich/Squeeze Theorem to compute $\lim_{x \to 0} x^2 \cos \left( \frac{1}{x} \right)$.

(c) Compute $\lim_{n \to \infty} \frac{n + (-1)^n}{n^2}$.
3. (a) Compute the derivative of \( f(x) = \frac{\ln(x)}{2 + x} \).

(b) Let \( g(x) = \frac{x}{(x + 1)^2}, \ x \neq -1. \) We know: \( g'(x) = \frac{1 - x}{(x + 1)^3} \quad g''(x) = \frac{2(x - 2)}{(x + 1)^4}. \)

(i) Where is \( g(x) \) increasing? Where is \( g(x) \) decreasing?

(ii) Where is \( g(x) \) concave up? Where is \( g(x) \) concave down?

(iii) Where are the critical and inflection points for \( g(x) \)? Classify them.
4. (a) Nina has one more gift to wrap for her family. Her gift is rather small, but it has a very recognizable shape so she wants to keep her family guessing by using the largest possible box. She doesn’t have any boxes left. It is late on Christmas Eve and all the stores are closed. She decides to make a box (with no lid) out of a 40 cm by 40 cm square sheet of cardboard. She cuts a square from each corner of the sheet so that she can fold up the sides to make her box.

What is the volume of the largest possible box that Nina can make?
What’s the length of the side of the square she cuts from each corner?

(b) Find the linear approximation of \( f(x) = \sin(2x) \cos(x) \) at \( x = \pi \).

Use this approximation to estimate the value of \( f(\pi + 0.1) \).
5. (a) Sodium chlorate crystals are easy to grow in the shape of cubes by allowing a solution of water and sodium chlorate to evaporate slowly.

(i) If $V$ is the volume of such a cube with side length $x$, calculate $dV/dx$ when $x = 3$ mm.

(ii) Show that the rate of change of the volume of a cube with respect to its edge length is equal to half the surface area of the cube.

(b) Consider the curve (pictured below) given by the equation $y^2 = \cos(x^2 - y)$.

Use implicit differentiation to compute $\frac{dy}{dx}$.

pts: /10
6. (a) Find both fixed points for the recursive sequence: \( a_{n+1} = \frac{a_n^2 + 8}{3a_n} \).

(b) What does the Stability Criterion say about the fixed points found in (a)?

It is given that \( f'(x) = \frac{x^2 - 8}{3x^2} \) is the derivative of \( f(x) = \frac{x^2 + 8}{3x} \).

(c) Given the initial value \( a_0 = 1.3 \), compute \( a_1, a_2, a_3, a_4, \) and \( a_5 \):

\[
\begin{align*}
a_1 &= \underline{\phantom{0}} \\
a_2 &= \underline{\phantom{0}} \\
a_3 &= \underline{\phantom{0}} \\
a_4 &= \underline{\phantom{0}} \\
a_5 &= \underline{\phantom{0}}
\end{align*}
\]

(d) Sketch a cobweb graph starting at \( a_0 = 1.3 \) on the given plot.

Make sure to mark at least the values \( a_1, a_2, a_3, a_4, \) and \( a_5 \) found in (c).

Use it to determine \( \lim_{n \to \infty} a_n \).
7. Use l’Hospital’s rule to evaluate the following limits:

(a) \( \lim_{x \to 2} \frac{8x - 12 - x^2}{x^2 - 4} \)

(b) \( \lim_{x \to 0^+} x^2 \cdot \ln x \)

(c) \( \lim_{x \to 0} \frac{\int_0^x e^h \, dh}{x^2} \)
8. (a) Compute the indefinite integral \( \int (1 + x^3) \sqrt{x} \, dx \)

(b) Solve the initial value problem

\[
\frac{dy}{dx} = \frac{e^{-x} + e^{x}}{2} \quad \text{with} \quad y = 1 \quad \text{when} \quad x = 0.
\]
9. (a) Consider the function \( f(x) = x^2 - 2x \) defined on the interval \([0, 3]\).

Estimate \( \int_{0}^{3} (x^2 - 2x) \, dx \) using right endpoints for \( n = 6 \) approximating rectangles all having bases of the same length, as shown in the picture.

(b) Use the Fundamental Theorem of Calculus to evaluate the definite integrals:

\[
\int_{0}^{2} (x^2 - 2x) \, dx = \quad \int_{2}^{3} (x^2 - 2x) \, dx = \quad \int_{0}^{3} (x^2 - 2x) \, dx =
\]
10. Use the Fundamental Theorem of Calculus to evaluate the following:

(a) If \( F(x) = \int_{0}^{3x^2+x} (1 + te^t) \, dt \) then the derivative \( F'(x) \) equals:

(b) Suppose \( h \) is a continuous function such that

\[
\begin{align*}
    h(1) &= -2 & h'(1) &= 2 & h''(1) &= 2 \\
    h(2) &= 6 & h'(2) &= 5 & h''(2) &= 13
\end{align*}
\]

and \( h'' \) is continuous everywhere.

Evaluate: \( \int_{1}^{2} h''(t) \, dt. \)
**Bonus.** (a) **Dialysis treatment** removes urea and other waste products from patients’ blood by diverting some of their bloodstream externally through a machine (dialyzer). The rate at which urea is removed from the blood (in mg/min) is often described by

\[ u(t) = \frac{r}{V} C_0 e^{-r V t} \]

where \( r \) is the rate of flow of blood through the dialyzer (in mL/min), \( V \) is the volume of the patient’s blood (in mL), and \( C_0 \) is the amount of urea in the blood (in mg) at time \( t = 0 \).

Evaluate the integral \( \int_0^{30} u(t) \, dt \).

What does this integral represent? (Note: \( r, V, \) and \( C_0 \) are constants.)

(b) Let \( g(x) = \int_0^x f(t) \, dt \), where \( f \) is the function whose graph is shown below.

(i) Determine the following values of this new function \( g \):

\[
\begin{align*}
g(0) &= \\
g(1) &= \\
g(2) &= \\
g(3) &= \\
g(4) &= \\
g(5) &= \\
\end{align*}
\]

(ii) Find the derivative of the function \( g \) and determine the \( x \) value at which \( g \) attains its maximum value.