1. (30) Suppose random variables \((X, Y)\) have a joint probability density function given by
\[
f(x, y) = c \ e^{-(2x + y)}; \quad \text{for } y \geq x \geq 0.
\]
(a) find the constant \(c\) that make \(f\) a density.
(b) find the two marginal p.d.f.s.
(c) find the joint p.d.f. of \((U, V)\) with
\[
U = X + Y, \quad V = Y - X.
\]
(d) find the marginal distribution or p.d.f. of \(V = Y - X\).
(e) find the conditional density of \(V\) given \(U\).

2. (25) Let \(X\) be a Poisson(\(\lambda\)) random variable. Show that, after proper standardization, as \(\lambda \to \infty\) we have \(X\) goes in distribution to a normal random variable.

3. (25) State and proof the Chebychev inequality.

4. (8) If \(X \sim N(2, 1)\) and \(Y \sim N(0, \sigma^2 = 4)\) and they are independent. Please compute \(\text{Var}(XY) = \?\).

5. (12) Suppose \(X_1, X_2, \cdots, X_n, \cdots\) is a sequence of exponential \((\lambda = 3)\) random variables. Show that
\[
\frac{X_n}{\sqrt{n}} \to 0
\]
almost surely as \(n \to \infty\).

**solution:** For any \(\epsilon > 0\), we compute \(P(|X_n/\sqrt{n}| > \epsilon) = P(X_n > \epsilon \sqrt{n}) = e^{-3\epsilon \sqrt{n}}\).
Since \(\sum_{n=1}^{\infty} e^{-3\epsilon \sqrt{n}} < \infty\) for any \(\epsilon > 0\), we have \(P(A_n \ i.o.) = 0\) where \(A_n = \{|X_n/\sqrt{n}| > \epsilon\}\).
This is the almost sure convergence we want.

**One last question to think about:** Show that
\[
\frac{\max_{1 \leq i \leq n} \{X_i\}}{\sqrt{n}} \to 0
\]
almost surely as \(n \to \infty\).