Example 19: The Kangaroo Lodge has 10 members \((A, B, C, D, E, F, G, H, I,\) and \(J)\). The club has five working committees: the Rules Committee \((A, C, D, E, I,\) and \(J)\), the Public Relations Committee \((B, C, D, H, I,\) and \(J)\), the Guest Speaker Committee \((A, D, E, F,\) and \(H)\), the New Year’s Eve Party Committee \((D, F, G, H,\) and \(I)\), and the Fund Raising Committee \((B, D, F, H,\) and \(J)\).

(a) Suppose we are interested in knowing which pairs of members are on the same committee. Draw a graph that models this situation. (Hint: Let the vertices of the graph represent the members.)

(b) Suppose we are interested in knowing which committees have members in common. Draw a graph that models this situation. (Hint: The vertices should not represent committee members.)
The sum of the degrees of all of the vertices of a graph equals twice the number of edges (and therefore is an even number.)

A graph always has an even number of odd vertices.

Example 20: Consider the graph with \( V = \{A, B, C, D, E\} \) and \( E = \{AD, AE, BC, BD, DD, DE\} \). Find the sum of the degrees of the vertices.

Example 21: Explain why in every graph the sum of the degrees of all the vertices equals twice the number of edges and why every graph must have an even number of vertices.

For a connected graph \( G \),

<table>
<thead>
<tr>
<th>Number of odd vertices</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4, 6, 8, \ldots</td>
<td></td>
</tr>
<tr>
<td>1, 3, 5, \ldots</td>
<td></td>
</tr>
</tbody>
</table>
An **algorithm** for a problem $X$ is a set of procedural rules that, when followed, always lead to a solution to $X$.

**Fleury’s algorithm** is an algorithm that finds an Euler circuit or an Euler path in a connected graph. In this algorithm, once we have traveled an edge, we can think of erasing it in our graph. We are left with the *yet to be traveled* part of the graph. We will only look at edges that are bridges in this part of the graph at each step of the algorithm.

The algorithm:
1. **Preliminaries:**
   - Make sure the graph is connected.
   - If you want to find an Euler circuit, make sure that there are no odd vertices.
   - If you want to find an Euler path, make sure that there are exactly two odd vertices.
2. **Start.** Choose an appropriate starting vertex.
3. **Intermediate Steps.** At each step, if you have a choice, *don’t choose a bridge of the yet-to-be-traveled part* of the graph. However, if you have only one choice, take the bridge.
4. **End.** When you can’t travel any more, the circuit or path is complete.

**Example 22:** Use Fleury’s algorithm to find an Euler circuit in the graph shown in Figure 1.
Example 23: Use Fleury’s algorithm to find an Euler path in the graph shown in Figure 2.