8 Graphs and Functions

Concepts:
- Graphs of Specific Functions
- The Domain and Range of a Function
- Piecewise Functions
- The Vertical Line Test

(Section 2.3)

8.1 The Graph of a Function

Definition 8.1
The graph of the function $f$ is the set of all points $(x, f(x))$ where $x$ is in the domain of $f$.

We can plot these points on an $xy$-Cartesian Coordinate System. You can see that the output values of the function (i.e., the $f(x)$ values) are the $y$-coordinates of the points.

You are responsible for graphs of basic functions.
You will need to know how to graph some basic functions without the help of your calculator.

- Linear Functions ($f(x) = mx + b$)
- Power Functions ($f(x) = x^n$) where $n$ is a positive integer.
- Square Root Function ($f(x) = \sqrt{x}$)
- Greatest Integer Function ($f(x) = \lfloor x \rfloor$)
- Absolute Value Function ($f(x) = |x|$)
- Piecewise-defined Functions.
Example 8.2
Sketch the graph of $g(x) = |x + 2|$.
Example 8.3 (Domain and Range from a Graph)
Find the domain and range of each of the functions graphed below.
Example 8.4 (Graphing Piecewise-defined Functions)

Sketch the graph of

\[ k(x) = \begin{cases} 
3x + 1 & \text{if } x \leq 2 \\
x^2 & \text{if } x > 2 
\end{cases} \]
Example 8.5 (Can you interpret the graph of a function?)
In the picture below, the graph of \( y = f(x) \) is the solid graph, and the graph of \( y = g(x) \) is the dashed graph. Find the true statement.

**Possibilities:**
(a) \( f(1) < g(1) \)
(b) \( g(2) = 2 \)
(c) \( f(-3) < g(-3) \)
(d) \( f(-1) > g(-1) \)
(e) \( f(-1) = 2 \)

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8.1.1 The Vertical Line Test

The **Vertical Line Test** is used to determine if \( y \) is a function of \( x \). If you have a graph on the Cartesian Coordinate System and at least one vertical line touches the graph in more than one place, then \( y \) is NOT a function of \( x \) for that graph. If the no vertical line touches the graph in more than one place, then \( y \) is a function of \( x \) for that graph.

Why does this work? The equation of a vertical line is of the form \( x = a \). If the vertical line \( x = a \) touches the graph in more than one place, then there is more than one output value (\( y \) value) that corresponds to the input value \( a \). Since there can only be one output value for every input value of a function, then \( y \) cannot be a function of \( x \).
Example 8.6 (Do you understand the vertical line test?)
Is $y$ a function of $x$ in the graph below?

Example 8.7 (Do you understand the vertical line test?)
Is $x$ a function of $y$ in the graph below?

What test determines if $x$ is a function of $y$?
8.2 Graphs of Commonly Used Functions

You should study these graphs and be able to reproduce them without using a graphing calculator.

Example 8.8 (The Power functions)
Note the lack of scale on these two graphs.

\[ f(x) = x^n \text{ for } n \text{ even} \]

\[ f(x) = x^n \text{ for } n \text{ odd} \]

Example 8.9 (The Square Root and Absolute Value Functions)

\[ f(x) = \sqrt{x} \]

\[ f(x) = |x| \]
Example 8.10 (The Greatest Integer Functions \( f(x) = \lfloor x \rfloor \))
Sketch the graph of the greatest integer function. This is an example of a step function.