9 Functions

Concepts:

- Obtain Graphs of Parametric Equations
- Graph Equations Where $x$ is Function of $y$
- Graphing Calculators and Parametric Equations

(Section 2.4)

9.1 Graphs in Cartesian Plane Defined by a Parameter

We saw in the last section that not all graphs represent functions. Parametric equations allow us to create graphs where $y$ is not necessarily a functions of $x$.

Example 9.1

The following graph would be difficult to describe using functional notation. Graphs like this appear frequently in Engineering and Science. We will see in later sections how to use the trigonometric functions and a parameter to create it.

![Graph diagram]
Using a third variable called a parameter usually denoted as $t$, we can express graphs in the Cartesian plane by allowing $x$ and $y$ to functions of $t$. Note that the parameter does not explicitly appear on the graph.

**Example 9.2**

Let $x = \frac{t^2}{3} - 4$ and $y = t - 1$. Fill in the table for the given values of $t$ and plot the resulting $x$ and $y$ values in the plane. Sketch the resulting graph as a curve.

<table>
<thead>
<tr>
<th>$t$</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Example 9.3
Let $x = t^3 - t$ and $y = 4 - t^2$. Fill in the table for the given values of $t$ and plot the resulting $x$ and $y$ values in the plane.

<table>
<thead>
<tr>
<th>$t$</th>
<th>-2</th>
<th>-1.5</th>
<th>-1</th>
<th>-.5</th>
<th>0</th>
<th>.5</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y$</td>
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</tr>
</tbody>
</table>
Example 9.4
Let \( x = t^3 - t \) and \( y = 4 - t^2 \). Use your graphing calculator to plot the curve.
9.2 Applications

A particle is moving through the Cartesian plane. Its position as a function of time is given parametrically by $x(t) = 5\sin(t)$ and $y(t) = 3\cos(t)$. Plot the trajectory for $t$ from 0 to $4\pi$ in steps of 0.1 seconds. Indicate where the particle is by labeling points on the graph when $t = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}, \frac{3\pi}{2},$ and $2\pi$. It will be helpful to use the parametric and table feature of your calculator.