10 Graphs and Transformations

Concepts:

- Graphs of Functions
- Sketching Graphs of Functions
- Applying Transformations to the Graph of a Function
- How Does a Graph Transformation Move a Point on a Graph?

(Section 2.5)

10.1 Graph Transformations

The graphs shown below have the same basic shape, but they are different. How?

These graphs have the same basic shape, but they have been transformed by shifts, scaling factors, and reflections. In this section, we will learn how simple modifications of a function by a constant can change the graph of the function. We first consider how a simple modification affects one point on the graph of a function.
Example 10.1 (Do you understand graph transformations?)
Suppose that the graph of $f$ contains the point $(2, 3)$. Find a point that must be on the graph of $g$. Explain how you had to move the point on the original graph $f$ to obtain a point on the new graph $g$.

- The graph of $g(x) = f(x) + 5$ must contain the point __________.

- The graph of $g(x) = f(x + 5)$ must contain the point __________.

- The graph of $g(x) = f(x) - 5$ must contain the point __________.

- The graph of $g(x) = f(x - 5)$ must contain the point __________.

- The graph of $g(x) = 5f(x)$ must contain the point __________.

- The graph of $g(x) = f(5x)$ must contain the point __________.
• The graph of \( g(x) = \frac{1}{5} f(x) \) must contain the point ____________.

• The graph of \( g(x) = f \left( \frac{1}{5} x \right) \) must contain the point ____________.

• The graph of \( g(x) = f(7x) + 5 \) must contain the point ____________.

• The graph of \( g(x) = 3f(7x + 1) + 5 \) must contain the point ____________.

• **A Challenge:** The graph of \( g(x) = 3(f(7(x + 1)) + 5) \) must contain the point ____________.
Example 10.2 (Do you understand graph transformations?)
Suppose that the graph of \( f \) contains the point \((a, b)\). Find a point that must be on the graph of \( g \). Also describe how you could transform the graph of \( f \) to obtain the graph of \( g \).

- The graph of \( g(x) = f(x) + d \) must contain the point

- The graph of \( g(x) = f(x + c) \) must contain the point

- The graph of \( g(x) = f(x) - d \) must contain the point

- The graph of \( g(x) = f(x - c) \) must contain the point

- The graph of \( g(x) = -f(x) \) must contain the point

- The graph of \( g(x) = f(-x) \) must contain the point

- The graph of \( g(x) = df(x) \) must contain the point

- The graph of \( g(x) = f(cx) \) must contain the point
Example 10.3 (Do you understand graph transformations?)
Let $f(x) = |x|$. Write $g(x)$ in terms of $f(x)$ and explain how you would transform the graph of $f$ to draw the graph of $g$. Sketch the graph of $g$.

- $g(x) = |x + 3| - 4$.

- $g(x) = 2|x| + 5$.

- $g(x) = 2(|x| + 5)$.

- $g(x) = |2x - 4|$. 
Example 10.4 (Do you understand graph transformations?)
In the picture below, the graph of \( y = f(x) \) is the solid graph, and the graph of \( y = g(x) \) is the dashed graph. Find a formula for \( g(x) \).

Possibilities:
(a) \( g(x) = f(x - 3) \)
(b) \( g(x) = f(x) - 3 \)
(c) \( g(x) = -3f(x) \)
(d) \( g(x) = f(x) + 3 \)
(e) \( g(x) = f(x + 3) \)
Example 10.5 (Do you understand graph transformations?)
In the picture below, the graph of \( y = f(x) \) is the solid graph, and the graph of \( y = g(x) \) is the dashed graph. Find a formula for \( g(x) \).

Possibilities:
(a) \( g(x) = -\frac{1}{4}f(x) \)
(b) \( g(x) = -4f(x) \)
(c) \( g(x) = \frac{1}{4}f(x) \)
(d) \( g(x) = 4f(x) \)
(e) \( g(x) = f(4x) \)