11 Functions

Concepts:
- Operations on Functions
- The Domain of a Composition of Functions.

(Section 2.6)

11.1 Operations on Functions

There are five basic operations on functions. You can find the sum, difference, product, quotient, or composition of functions. The first four operations are straightforward, and you should read about them in your textbook. We will examine function composition in a bit more detail.

The machine diagram for the function composition $f \circ g(x) = f(g(x))$ is shown below.

How do you determine the domain of $f(g(x))$?
- $x$ must be in the domain of $g$.
- $g(x)$ must be in the domain of $f$. 
Example 11.1 (Function Composition and Domain)

Let \( f(x) = \frac{1}{x-2} \) and \( g(x) = \sqrt{x+1} \).

- Find \( f(g(x)) \).
- Find the domain of \( f(g(x)) \).
Example 11.2 (Function Composition and Domain)
Let $f(x) = \frac{1}{x - 2}$ and $g(x) = \sqrt{x + 1}$.

- Find $g(f(x))$.
- Find the domain of $g(f(x))$.

Example 11.3 (Function Composition and Domain)
Let $f(x) = \frac{1}{x}$.

- Find $f(f(x))$.
- Find the domain of $f(f(x))$. 
Example 11.4 (Do You Understand Function Composition?)
Let $f(x) = 5x + 2$ and $g(x) = \frac{x - 2}{5}$.

- Find $f(g(x))$.
- Find $g(f(x))$.

Example 11.5 (Do You Understand Function Composition?)
In the picture below, the graph of $y = f(x)$ is the solid graph, and the graph of $y = g(x)$ is the dashed graph. Use the graphs to evaluate $f(g(-2))$.

Possibilities:
(a) 0
(b) 2
(c) 4
(d) -3
(e) 1
Example 11.6 (An Application of Function Composition)
A rock is dropped into a pond creating circular ripples. The radius of the outer circle is increasing at a rate of 3 feet per second. Express the area enclosed by the outer circle as a function of time.

- Why is this question in a section about function composition?

- Express the radius as a function of time.

- Express the area as a function of the radius.

- How do these functions relate to the first function you found in this example?
Example 11.7 (An Application of Function Composition)
You have a $5 coupon from the manufacturer good for the purchase of a new cell phone. Your cell provider is also offering a 15% discount on any new phone. You make two trips to cell phone stores to look at various phones. On your first trip, you speak with Mandy. Mandy tells you that you can take advantage of both the coupon and the discount. She will apply the discount and then apply the coupon to the reduced price. On your second trip, you talk to Kelly. She also says that you can take advantage of both deals, but she tells you that she will apply the coupon and then apply the discount.

Let $x$ represent the original sticker price of the cell phone.

- Suppose that only the 15% discount applies. Find a function $f$ that models the purchase price of the cell phone as a function of the sticker price $x$.

- Suppose that only the $5 coupon applies. Find a function $g$ that models the purchase price of the cell phone as a function of the sticker price $x$.

- If you can take advantage of both deals, then the price you will pay is either $f(g(x))$ or $g(f(x))$, depending on the order in which the coupon and the discount are applied to the price. Find $f(g(x))$ and $g(f(x))$.

- The price that Mandy is offering you is modeled by $\underline{\hspace{2cm}}$.

- The price that Kelly is offering you is modeled by $\underline{\hspace{2cm}}$. 
Example 11.8 (An Application of Function Composition)

Ed and Carol leave the house at 9:00 AM. Ed travels due north at a rate of 40 mph and Carol travels due east at a rate of 30 mph.

- Functions can have more than one input. Express the distance between Ed and Carol as a function of both $E$ and $C$, where $E$ is the distance between Ed and the house and $C$ is the distance between Carol and the house.
- Express $E$ as a function of the time $t$ in hours since 9:00AM.
- Express $C$ as a function of the time $t$ in hours since 9:00AM.
- Express the distance between Ed and Carol as a function of time.
Example 11.9 (An Application of Function Composition)
Joni is leaning against a lamppost that is 20 feet high. She begins walking away from the lamppost. Joni is 5 feet tall and she is walking at a rate of 6 feet per second. Express the length of Joni’s shadow as a function of time.