27 Some Trigonometric Identities

Concepts:
- Trigonometric Identities
  - Pythagorean Identities
  - Periodicity Identities
  - Negative Angle Identities

(Section 6.2)

27.1 Some Identities

27.1.1 The Pythagorean Identities

Theorem 27.1 (The Pythagorean Identity for $\sin$ and $\cos$)

For every real number $t$,

$$\sin^2(t) + \cos^2(t) = 1$$

PROOF:
Example 27.2
If $\frac{\pi}{2} < t < \pi$, and $\cos(t) = \frac{-4}{7}$, find exact values for the other trigonometric functions at $t$.

Example 27.3
Find two other Pythagorean Identities that involve the other trigonometric functions.
27.1.2 Periodicity Identities

Think about the unit circle as you discover the periodicity identities. It might also help to think of $\tan(\theta)$ as the slope of the terminal side of the angle $\theta$. Why?

**Theorem 27.4**

- $\sin(x \pm \theta) = \sin(x)$
- $\cos(x \pm \theta) = \cos(x)$
- $\tan(x \pm \theta) = \tan(x)$
- $\csc(x \pm \theta) = \csc(x)$
- $\sec(x \pm \theta) = \sec(x)$
- $\cot(x \pm \theta) = \cot(x)$

27.1.3 Negative Angle Identities

Think about the unit circle as you discover the periodicity identities.

**Theorem 27.5 (Negative Angle Identities)**

- $\sin(-x) =$
- $\cos(-x) =$
- $\tan(-x) =$
- $\csc(-x) =$
- $\sec(-x) =$
- $\cot(-x) =$