28 Trigonometric Graphs

Concepts:
- Period
- The Graph of the sin, cos, tan, csc, sec, and cot Functions
- Applying Graph Transformations to the Graphs of the sin, cos, tan, csc, sec, and cot Functions
- Using Graphical Evidence to Make Conjectures about Identities
- The Periodicity Identities
- The Relationship Between the Periodicity Identities and the Periods of the Trigonometric Graphs

(Sections 6.3, 6.4)

28.1 Period

Intuitively, the graph of a function with period \( k \) is a graph that does not change when you shift the graph to the right or left \( k \) units. More formally,

**Definition 28.1**
The graph of \( f(x) \) is **periodic with period** \( k \) if

\[
    f(x + k) = f(x)
\]

and

\[
    f(x - k) = f(x).
\]

**Example 28.2**
Find the period the following graph.

![Graph Image](image-url)
28.2 The Graph of $y = \sin(x)$

Recall the unit circle definition for $\sin(x)$. Let’s see how we can use the unit circle to sketch the graph of $y = \sin(x)$.

The Graph of $y = \sin(x)$:

Period:

Corresponding Periodicity Identity:
Example 28.3
List the transformations needed to convert the graph of $y = \sin(x)$ into the graph of $y = 3\sin(x + \frac{\pi}{2})$. What is the period of the graph? Sketch the graph.

Example 28.4
List the transformations needed to convert the graph of $y = \sin(x)$ into the graph of $y = \sin(\frac{x}{2})$. What is the period of the graph? Sketch the graph.

Example 28.5
List the transformations needed to convert the graph of $y = \sin(x)$ into the graph of $y = A\sin(bx + c)$. What is the period of the graph?
28.3 The graph of $y = \cos(x)$

The Graph of $y = \cos(x)$:

Period:

Corresponding Periodicity Identity:
Example 28.6
List the transformations needed to convert the graph of \( y = \cos(x) \) into the graph of \( y = \cos(\frac{2x}{4}) \). What is the period of the graph? Sketch the graph.

Example 28.7
Use graphical evidence to determine which of the following MIGHT be trigonometric identities and which definitely cannot be trigonometric identities.

- \( \sin(x + \frac{\pi}{2}) = \cos(x) \)
- \( \sin(2x) = \sin(x) \cos(x) \)

Example 28.8
In the example above, we saw that \( \sin(2x) \neq \sin(x) \cos(x) \). Can you modify the original equation to find a new equation that has the possibility of being an identity?

Example 28.9
This example is taken from Exercise 61 in Section 6.4 of your textbook. If you have a TI-83 or TI-84 calculator, look at the graphs of \( y = \cos(x) \) and \( y = 1 \) in the viewing window \([0, 188\pi] \times [-3, 3]\). Trace the graph of \( y = \cos(x) \). Why do you think the calculator displays the wrong graph? What are you supposed to learn from this example?
28.4 The graph of \( y = \tan(x) \)

Let \( x \) be an angle in standard position. Then the terminal side of that angle intersects the unit circle at the point \( (a, b) \). What is the slope of the terminal side of the angle?

To understand the graph of the tangent function, it helps to think of the terminal side of an angle sweeping around the unit circle.

Period:

Corresponding Periodicity Identity:
28.5 An Application

Example 28.10 (Example 9 from Section 6.5 of your textbook)

Katy’s blood pressure can be modeled by the function

\[ f(t) = 22 \cos(2.5\pi t) + 95, \]

where \( t \) is the time (in seconds) and \( f(t) \) is in millimeters of mercury. The highest pressure (systolic) occurs when the heart beats, and the lowest pressure (diastolic) occurs when the heart is at rest between beats. The blood pressure is the ratio systolic/diastolic.

(a) Graph the blood pressure function over a period of two seconds and determine Katy’s blood pressure.

(b) Find Katy’s pulse rate (number of heartbeats per minute).

Example 28.11

(a) Jake has a blood pressure of 140/80. Assume that Jake’s heart beats once every second. Find a model for Jake’s blood pressure.

(b) Joni has a blood pressure of 120/70. Assume that Joni’s heart rate is 80 beats per minute. Find a model for Joni’s blood pressure.