Instructions: Answer the questions in the space provided. Show all work to receive partial credit if any. Unsupported work will receive no credit. Circle final answer. This quiz is worth 10 points.

1. (3 points) Use the definition of the Laplace transform and improper integrals to show

\[ \mathcal{L}\{e^{at}\} = \frac{1}{s-a}, \quad s > a. \]

\[ \int_0^{\infty} e^{at} e^{-st} \, dt = \int_0^{\infty} e^{(s-a)t} \, dt = \frac{1}{s-a} \lim_{a \to \infty} \int_0^{\infty} e^{(s-a)t} \, dt \]

\[ = \frac{1}{s-a} \quad \text{as} \quad a \to \infty \]

\[ = \frac{1}{s-a} \quad \text{as} \quad s-a > 0 \]

2. (3 points) Find the inverse Laplace transform of each \(F(s)\).

(a) \( \frac{s}{s^2 + 1} \)

\[ \mathcal{L}^{-1}\{ \frac{s}{s^2 + 1} \} = \cos t \]

(b) \( \frac{1}{s^2 - 2s + 2} \)

\[ \mathcal{L}^{-1}\{ \frac{1}{s^2 - 2s + 2} \} = \frac{1}{(s-1)^2 + 1} \]

3. (4 points) Apply the Laplace transform to the differential equation and solve for \( \mathcal{L}\{y\} \). Do not find the inverse transform. What does \( u_2(t)(t-2) \) in the equation represent?

\( y'' + 3y' + y = t + u_2(t)(t-2), \quad y(0) = 0, \quad y'(0) = 1. \)

\[ s^2 \mathcal{L}\{y\} - sy(0) - y'(0) + 3(\mathcal{L}\{y\} - y(0)) + \mathcal{L}\{y\} = \frac{1}{s^2} + e^{rac{1}{s^2}} \]

\[ \mathcal{L}\{y\} = \frac{1}{s^2 + 3s + 1} \]

\[ \mathcal{L}\{y\} = \frac{1}{s^2 + 3s + 1} \]

\( u_2(t)(t-2) \) is a shifted to the right by 2 units and switched on at \( t = 2 \).