MA110 Daily Quiz 1

August 26, 2015
Suppose \( x \) is a real number. According to the definition of absolute value, \( |x| \) is 

\[
|x| = \begin{cases} 
  x & \text{if } x > 0 \\
  -x & \text{if } x \leq 0 
\end{cases}
\]

**Answer:** \( |x| = \begin{cases} 
  x & \text{if } x > 0 \\
  -x & \text{if } x \leq 0 
\end{cases} \).

2. If \( A = (3, 14) \) and \( B = (8, 2) \) are points in the plane. Use the distance formula to calculate the distance from \( A \) to \( B \) is exactly.

**Answer:** \( \sqrt{(8 - 3)^2 + (2 - 14)^2} = 13. \)
Quiz 3 (Numbers)

MA110 Daily Quiz 3

August 31, 2015
1. Use the axioms to explain why \( a \times 0 = 0 \) for any number \( a \).

**Answer:**

\[
\begin{align*}
0 + 0 &= 0 \\
(0 + 0) + (0 + 0) &= (a \times 0) + (a \times 0) \\
(a \times 0 + a \times 0) + (a \times 0 + a \times 0) &= (a \times 0 + a \times 0) + (a \times 0 + a \times 0) \\
a \times 0 + (a \times 0 + a \times 0) &= a \times 0 + (a \times 0 + a \times 0) \\
a \times 0 + (0) &= 0 \\
a \times 0 &= 0
\end{align*}
\]

2. Write down the decimal expansion for a number that is not rational and explain why the number is not rational.

**Answer:**

Let \( x = .11010010001000010000001000000010000000010000000001 \cdots \)

This begins with 1 followed by 0 zeros, then 1 followed by 1 zeros, then 2 zeros, etc. This cannot be rational since rational numbers have repeating decimals. If this repeated in blocks of \( n \) digits then a complete pattern would fall in any string of length at least \( 2n \) zeros. So the block would be all zeroes which would make the decimal expansion 0 from some point on. That is a contradiction since there are infinitely many 1’s in the expansion.
Quiz 4 (Numbers) Ans

MA110 Daily Quiz 4 Ans

Sept 2, 2015
(1) What are the quotient and remainder that would be produced if we were to divide $7^{100} + 17$ by 7 using long division? Justify your answer.

**Answer:** The quotient $q$ and remainder $r$ are the unique integers which satisfy $7^{100} + 17 = 7q + r$ with $0 \leq r < 7$.

We observe that $N = 7^{100} + 17 = 7^{100} + 14 + 3 = 7^{100} + (2)7 + 3 = 7(7^{99} + 2) + 3$. So since $0 \leq 3 < 7$, 3 is the remainder.

(2) What is the remainder if 123,456,789 is divided by 1,000? Justify your answer.

(Note that this a much simpler version of the one on the problems list.)

**Answer:** 123,456,789 = (123,456) * 1000 + 789 Since $0 \leq 789 < 1,000$ the remainder is 789.
1) Determine whether

\[
\frac{12399}{68799} = \frac{12299}{67899}.
\]

Show your work. Your solution cannot involve decimal numbers. Multiplying, adding, and subtracting integers with the calculator is ok.

**Answer:** \( \frac{a}{b} = \frac{c}{d} \) if and only if \( ad = bc \) so these are equal if and only if \( 12399 \times 67899 = 12299 \times 68799 \). This is \( 841879701 = 846158901 \) which is false so the two are not equal.

2) Determine whether

\[
\frac{3 + 2i}{5 + i} = \frac{5 - i}{4 + 6i}.
\]

Explain your answer!!

**Answer:** False because \( (3 + 2i)(4 + 6i) = 26i \neq (5 + i)(5 - i) = 26 \). 

(Quiz 5 ( Complex Numbers))
Quiz 6 (Linear Equations)

MA110 Daily Quiz 6 (Linear Equations) Ans

Sept 9, 2015
If $z = -3 + 4i$ then

\textbf{a)} A linear equation is \textbf{consistent} if

\begin{itemize}
  \item \textbf{Ans:} Ans: it has at least one solution.
\end{itemize}

\textbf{b)} A linear equation is \textbf{inconsistent} if

\begin{itemize}
  \item \textbf{Ans:} Ans: It has no solutions.
\end{itemize}

\textbf{c)} Explain why $(1, -1)$ is or is not a solution to the system of linear equations

$$S = \{ E_1 : x + 3y = -2, E_2 : 6x - 2y = 8 \}$$

\begin{itemize}
  \item \textbf{Ans:} Ans: $E_1 : (1) + 3(-1) = -2$ is true.
  \item $E_2 : 6(1) - 2(-1) = 8$ is true. It is a solution to every equation $S$ so it is a solution to $S$.
\end{itemize}
Quiz 7 (Expressions) ANS

MA110 Daily Quiz 7 (Expressions) ANS

Sept 11, 2015
Definition: An expression $M$ is a **monomial** in the variable $x$ if $M(x) = c \times x^n$ with $c$ a _________, and $n$ a _________. In this case $c$ is called the _________ of $M$ and $n$ the _________ of $M$.

**Ans:** Ans: number, non-negative integer, coefficient, degree

b) What are the degree and lead coefficient of $(x + 1)^3 - (x + 2)^3$. Justify your answers.
Degree = ___________. Lead coefficient = ___________.

**Ans:** 2, −3

c) If $f(x) = x^5 - x^4 - x - 1$ then the coefficient of $x^5$ in $(x - 2)f(x)$ is __________

**Ans:** $x^5(-2) + x(-x^4) = -3x^5$ so the coefficient is $-3$. 
Quiz 8 (Expressions) Ans

MA110 Daily Quiz 8 (Expressions) Ans

Sept 14, 2015
1. Suppose $t$ is a number such that $3 \ast (t - 7)^2 - 5 \ast (t - 7) = -2$. Then $t = \underline{\hspace{2cm}}$.

**Ans:** $x = t - 7$ then $3x^3 - 5x + 2 = 0$. By quadratic formula

$$x = \frac{-(-5 \pm \sqrt{(-5)^2 - 4 \ast 3 \ast 2})}{2 \ast 3}.$$ $x = 1, \frac{2}{3}$, $x = x + 7 = 8, 7\frac{2}{3}$

2. If $(3y - 2x)^5$ is expanded and simplified then the coefficient of $x^2y^3$ will be $\underline{\hspace{2cm}}$. You may leave your answer in factored form. (Show your work!)

**Ans:** $C(5, 3)(3y)^3(-2x)^2 = (10)3^3(-2)^2x^2y^3$
Quiz 9 (Linear Equations) ANS

MA110 Daily Quiz 9 (Linear Equations) ANS

Sept 16, 2015
1 Solve \( \begin{cases} 4x + 10y = 9 \\ -3x - 4y = 1 \end{cases} \) for \( x \) using Cramer’s Rule. You must make it clear that you have used Cramer’s rule.

Ans: \( D_x = \det(\begin{vmatrix} 9 & 10 \\ 1 & -4 \end{vmatrix}) \), \( D = \det(\begin{vmatrix} 4 & 10 \\ -3 & -4 \end{vmatrix}) \)

\[ x = \frac{D_x}{D} = \frac{-46}{14} \]
2. The height and width of a rectangle are unknown. When 12 exact copies are arranged in 4 stacks of three to make a larger rectangle then the perimeter of the larger rectangle is 80 units. When that same 12 are arranged in 6 stacks of 2 then the perimeter of the larger rectangle is 100 units.

What are the dimensions of the rectangle?

width = __________, height = __________.

Ans: $eq1 : 8w + 6h = 80$, $eq2 : 12w + 4h = 100$,

$\frac{1}{8}eq1 : w + \frac{6}{8}h = 10$, $w = 10 - \frac{3}{4}h$.

$eq2 : 12(10 - \frac{3}{4}h) + 4h = 100$, $120 - 9h + 4h = 100$, $20 = 5h$, $h = 4$, $w = 10 - \frac{3}{4}4 = 7$. 

(MA110 Daily Quiz 9 (Linear Equat: Quiz 9}
The height and width of a rectangle are unknown. When 12 exact copies are arranged in 4 stacks of three to make a larger rectangle then the perimeter of the larger rectangle is 80 units. When that same 12 are arranged in 6 stacks of 2 then the perimeter of the larger rectangle is 100 units.

What are the dimensions of the rectangle?

width = __________, height = __________.

Ans: \( eq1 : 8w + 6h = 80, \) \( eq2 : 12w + 4h = 100, \) 
\( \frac{1}{8}eq1 : w + \frac{6}{8}h = 10, \) \( w = 10 - \frac{3}{4}h. \)
\( eq2 : 12(10 - \frac{3}{4}h) + 4h = 100, 120 - 9h + 4h = 100, 20 = 5h, \) 
\( h = 4, \) \( w = 10 - \frac{3}{4}4 = 7. \)
1. What is the difference between a **number** and a **numeral**?

**Answer:** A numeral is a name for a number.

2. Assume that $3 = 4$ and prove that you are the president of UK.

**Answer:** If $3 = 4$ then subtracting 2 from both sides we get $1 = 2$. The president of UK and you are 2 people but $2 = 1$ so the president of UK and you are one person. Therefore you are the president of UK.
Quiz 10 (Integers) Ans

MA110 Daily Quiz 10 (Integers) Ans

Sept 18, 2015
Use the Aryabhata algorithm to express the fraction $\frac{9723}{10186}$ as the ratio of two positive integers with no common factor (i.e. in “lowest terms”).

Ans: From

\[
\begin{array}{ccc|c|c|c}
& 1 & 0 & 1 & 10186 \\
-1 & 0 & 1 & 9723 \\
-21 & 1 & -1 & 463 \\
-21 & 22 & 0 & \\
\end{array}
\]

we see that

\[
21 \times 10186 = 22 \times 9723 \quad \text{so} \quad \frac{21}{22} = \frac{9723}{10186}
\]
Quiz 10 (Integers) Ans

MA110 Daily Quiz 10 (Integers) Ans

Sept 18, 2015
Use the Aryabhata algorithm to express the fraction \( \frac{9723}{10186} \) as the ratio of two positive integers with no common factor (i.e. in "lowest terms").

**Ans:** From

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<td>0</td>
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we see that

\[
21 \times 10186 = 22 \times 9723 \text{ so } \frac{21}{22} = \frac{9723}{10186}
\]
Quiz 11 (Ans)

MA110 Daily Quiz 11 (Ans)

Sept 21, 2015
a) Write \( \frac{x - 3}{x + 3} - \frac{x - 1}{x + 1} \) in the form \( \frac{A}{B} \) where \( A \) and \( B \) are polynomials. Show your work.

\[ \text{Ans: } \frac{-4x}{(x+1)(x+3)} \]

b) If \( 3x^2 y^7 \) is regarded as a polynomial in \( y \) with coefficients polynomials in \( x \) then its degree is \( \underline{\text{________}} \) and its coefficient is \( \underline{\text{________}} \).

\[ \text{Ans: } 7, \, 3x^2 \]

c) Give the proper definition of the absolute value, \( |x| = \)

\[ \text{Ans: } |x| = \begin{cases} 
  x & \text{if } x \geq 0 \\
  -x & \text{if } x < 0 
\end{cases} \]
Quiz 12

MA110 Daily Quiz 12 (Ans)

Sept 23, 2015
a) Find the value of $w$ so that the determinant of the matrix $\begin{vmatrix} -6 & w \\ 3 & 5 \end{vmatrix}$ is 0. Show your work.

\textbf{Ans:} $-6(5) - 3w = 0$, $w = -10$

b) A triangle and a 5 by 8 rectangle are cut out of a square of unknown side $x$, leaving the shaded region indicated in the diagram. Express the area $A$ of the region as a $A(x)$, a polynomial in $x$.

\textbf{Ans:} $A(x) = \frac{1}{2}x^2 - 40$
The line $M$ has two sets of coordinates “$z$” and “$w$” where the $z$ and $w$ coordinates of a point are related by the formula

$$w = az + b$$

- The point $P$ has $z$ coordinate 0 and $w$ coordinate 5.
- The point $Q$ has $z$ coordinate 1 and $w$ coordinate is 3. The formula for translating from $z$ to $w$ coordinates is $w = \_\_\_\_\_\_\_\_\_\_\_. The point $S$ whose $w$ coordinate is $-1$ has $z$ coordinate \_\_\_\_\_\_\_.

**Ans:** $P : 5 = a(0) + b, Q : 3 = a(1) + b$ give $w = -2z + 5, 3$
Quiz 14 (Coordinates)

MA110 Daily Quiz 14 (Coordinates)

Sept 28, 2015
Suppose we have \((x, y)\) and \((z, w)\) coordinates for points in the plane. If the formulas
\[
\{ z = -x + 2, \quad w = y - 3 \}
\]
define the transformation from \((x, y)\) to \((z, w)\) coordinates, then the formulas defining the reverse transformation from \((z, w)\) to \((x, y)\) coordinates are:
\[
x = \quad , \quad y = \quad .
\]

(b) If \(O_{x,y}\) is the origin in the \(x - y\) coordinate system then what are the \(z - w\) coordinates for \(O_{x,y}\)?

Ans: \(x = -z + 2, \quad y = w + 3\) (b)
\[
z = -(0) + 2, \quad w = 0 - 3 \quad \text{so the coordinates are} \quad (2, -3)\]
Quiz 15 (Coordinates) (Ans)

MA110 Daily Quiz 15 (Coordinates) (Ans)

Sept 30, 2015
A transformation $T$ maps the plane to the plane by multiplication by a complex number. That is, we view the point $P(x, y)$ as the complex number $P = x + iy$ and there is a complex number $C = a + bi$ such that $T(P) = CP$.

The transformation takes the point $A$ whose coordinates are $(5, 2)$ to the point $T(A)$ whose coordinates are $(3, 7)$

a.) The complex number $C = a + bi$ is _________

b.) The effect of $T$ is to rotate each point $P$ by $\arctan\left(\frac{b}{a}\right)$ degrees counterclockwise about the origin (in this case the angle is 45 degrees) and to stretch its direction from the origin by a factor of _________.

**Ans:** (a) $\frac{3 + 7i}{5 + 2i} = 1 + i$, (b) $|1 + i| = \sqrt{2}$
Quiz 16 (lines) (Ans)

MA110 Daily Quiz 16 (lines) (Ans)

Oct 2, 2015
Quiz 16 (lines) (Ans)

MA110 Daily Quiz 16 (lines) (Ans)

Oct 2, 2015
Determine parametric equations \( x(t) \) and \( y(t) \) for the line through the points \( A(4, 7) \) and \( B(-5, 9) \) such that the point \( A \) corresponds to \( t = 0 \) and \( B \) corresponds to \( t = 1 \).

**Ans:** 
\[
P(t) = A + t(B - A) = (4, 7) + t(-9, 2), x(t) = 4 + t(-9), y(t) = 7 + 2t
\]

The line \( L \) is given parametrically by 
\[
x(t) = 3t - 5, y(t) = -2t + 7. \]
The cartesian slope-intercept equation \( (y = mx + b) \) for \( L \) is

**Ans:** 
\[
y = -\frac{2}{3}x + \frac{11}{3}
\]
Quiz 17 (Lines) (Ans)

MA110 Daily Quiz 17 (Lines) (Ans)

Oct 5, 2015
Suppose $A = (1, 3)$, $B = (7, 6)$, and $C = (2, 7)$. One set of parametric equations for the line through $C$ which is parallel to the line through $A$ and $B$ is

**Ans:** $P(t) = C + t(B - A) = (2, 7) + t(6, 3) = (2 + 6t, 7 + 3t)$ or $\{x(t) = 2 + 6t, y(t) = 7 + 3t\}$. 
a) What is the point \( P(x, y) \) at the intersection of the line \( L \) which has parametric equations \( x(t) = 2t, y(t) = t - 1 \) and the line \( M \) which has parametric equations \( x(s) = -s + 1, y(s) = 5s \)?

**Ans:** \( \{2t = -s + 1, t - 1 = 5s\} \), yields \( s = \frac{-1}{11}, t = \frac{6}{11} \). Use either value, e.g, \( x = 2 \left( \frac{6}{11} \right), y = \frac{6}{11} - 1, P = (\frac{12}{11}, \frac{-5}{11}) \)

b) If \( A = (2, -5), B = (5, -1) \), find parametric equations for the perpendicular bisector of the line segment \( \overline{AB} \)

**Ans:** midpt = \( \frac{1}{2}(A + B) = (\frac{7}{2}, -3) \), direction (generalized slope) = \( B - A = (3, 4) \), normal direction \( = (B - A)^\perp = (-4, 3) \), \( (x(t), y(t)) = (\frac{7}{2}, -3) + t(-4, 3) \)
Quiz 19 (Lines) ANS

MA110 Daily Quiz 19 (Lines) ANS

Oct 9, 2015
Find the point of intersection of the line $L$ which passes through points $A(-5, 8)$ and $B(1, 2)$ and the line $M$ which passes through $C(-3, 2)$ and $D(4, 1)$.

**Ans:** $L$ has parametrization $P(t) = A + t(B - A)$ and $M$ $Q(s) = C + s(D - C)$. The point of intersection determined by the values of $t$ and $s$ for which $P(t) = Q(s)$ or 

$(-5, 8) + t((1, 2) - (-5, 8)) = (-3, 2) + s((4, 1) - (-3, 2))$

$(6t - 5, -6t + 8) = (7s - 3, -s + 2)$ this gives $6t - 6 = 7s - 3$

$-6t + 8 = -s + 2$

which have $s = \frac{2}{3}, s = \frac{10}{9}$ as the unique solution.

It checks that $P(\frac{10}{9}) = Q(\frac{2}{3}) = (\frac{5}{3}, \frac{4}{3})$
Quiz 20 (Lines) ANS

MA110 Daily Quiz 19 (Lines)

Oct 14, 2015
Each of the graph sketches below is that of a function of the form \( f : \mathbb{R} \to \mathbb{R} \) where \( f(x) = ax^2 + bx + c \) or \( f(x) = ax + b \). Complete the table below by entering “+”, “-”, “0”, or “\( \infty \)” to indicate the signs (or non-existence) of the coefficients or expressions for each of the graphs A-D.

<table>
<thead>
<tr>
<th></th>
<th>( a )</th>
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<th>( c )</th>
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<tr>
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Quiz 21 (Functions) ANS

Oct 16, 2015
1. **Definition:** The function $f : D \to T$ is **one-to-one (injective)** if · · ·  
   **Ans:** $f(a) = f(b)$ implies that $a = b$.  
   Alternately, if $a \neq b$ then $f(a) \neq f(b)$.

2. **Definition:** The function $f : D \to T$ is **onto (surjective)** if · · ·  
   **Ans:** for every $t \in T$ there is $d \in D$ such that $f(d) = t$.

3. **Definition:** $f : S \to \mathbb{R}$ has an **absolute maximum value** at $x = t$ if $F(x) \leq F(t)$ for every $x \in S$.

4. **Definition:** $f : S \to \mathbb{R}$ has an **absolute minimum value** at $x = t$ if $F(t) \leq F(x)$ for every $x \in S$.

5. **Definition:** $f : S \to \mathbb{R}$ has an **absolute extremum value** at $x = t$ if $f$ has either an absolute maximum or an absolute minimum at $x = t$.

6. The absolute extremum of the quadratic function $f(x) = ax^2 + bx + c$ is _______. If $a > 0$ then the function has an absolute __________ and its value is ___________. If $a < 0$ then the function has an absolute ___________ there and its value is ___________.  
   **Ans:** $-\frac{b}{2a}$, absolute minimum, $f(-\frac{b}{2a})$, absolute maximum, $f(-\frac{b}{2a})$. 
Quiz 22 (Review) ANS

MA110 Daily Quiz 22 (Review) ANS

Oct 19, 2015
The coordinates of the point $A$ are $(5, 4)$ and those of the point $B$ are $(9, 3)$. What are the coordinates of the point which is on the x-axis and such that the triangle $PAB$ is a right triangle with the right angle at the point $A$?

**Ans:** A parametrization of the line $L$ through $A$ and $B$ is $P(t) = A + t(B - A) = ((5, 4) + t((9, 3) - (5, 4))) = (5, 4) + t(4, -1)$. The perpendicular direction to $L$ is $(4, -1) \perp = (1, 4)$ and the line through $A$ with this direction is $Q(s) = (5, 4) + s(1, 4) = (5 + s, 4 + 4s) = (x(s), y(s))$. This line meets the $x$ axis when $y(s) = 0$ or $s = -1$ so the point $C$ is $(4, 0)$. Alternately one can set $C = (t, 0)$ then the slope of $AC = \frac{4-0}{5-t}$ and the slope of $AB = \frac{3-4}{9-5} = \frac{-1}{4}$. We want $AB$ and $AC$ perpendicular to their slopes must be negative reciprocals so we must have $-\frac{1}{\frac{-1}{4}} = \frac{4-0}{5-t}$ so

\[
4 = \frac{4}{5-t} \\
1 = \frac{1}{5-t} \\
5 - t = 1 \text{ or } t = 4
\]
1. If $f : D \to T$ is a function then $D$ is called the ____________, and $T$ the ____________ of $f$.

The range of $f$ is $\{______ \mid \__________\}$.

**Ans:** domain, target, $\{f(x) \mid x \in D\}$. 
Quiz 23 (Reflection) ANS

MA110 Daily Quiz 23 ANS

Oct 23, 2015
A laser beam passes through the points $A(-5, 2)$ and $B(-1, -1)$. It meets the line $L$ through $O(0, 0)$ and $C(1, 1)$ at the point $B$ and is reflected. Find parametric equations for the line $R$ along which the beam travels after it is reflected.

**Ans:** Let $M$ be the line through $A$ and $B$. Then a parametric equation for $M$ is $M(t) = A + t(B - A)$ so the direction of $M$ is $B - A = (4, -3)$.

The direction of $L$ is $(C - O) = (1, 1)$

We resolve the direction of $M$ into its component that is parallel to $L$ and its component that is parallel to $L$.

Write $(4, -3) = \alpha(C - O) + \beta(C - O)^\perp$

$(4, -3) = \alpha(1, 1) + \beta(-1, 1) = (\alpha - \beta, \alpha + \beta)$

$\alpha - \beta = 4$

$\alpha + \beta = -3$

$\alpha = \frac{1}{2}, \beta = -\frac{7}{2}$

The direction of $R$ is $\alpha(C - O) - \beta(C - O)^\perp$

$\frac{1}{2}(1, 1) - \frac{7}{2}(-1, 1) = (-3, 4)$

So a $R(s) = (-1, -1) + s(-3, 4)$
1. (5 points) If \( A = (-1, 4) \) and \( B = (6, 2) \) then \( P(t) = (-1, 4) + t(7, -2) \) is a parameterization of the line \( L \) through \( A \) and \( B \) such that \( P(0) = A \) and \( P(1) = B \). What value of the parameter \( t \) corresponds to the point on \( K \) on \( L \) which is between \( A \) and \( B \) and such that the distance from \( A \) to \( K \) is 7 times the distance from \( B \) to \( K \)?

Ans:
\[
|t| = 7|1 - t| \\
t = 7 - 7t, \quad 8t = 7 \quad t = \frac{7}{8} \quad \text{(inside)} \\
-t = 7 - 7t, \quad 6t = 7 \quad t = \frac{7}{6} \quad \text{(outside)}
\]

2. (5 points) Suppose \( f(x) = ax^2 + bx + c \). Sketch possible graphs \( A \) and \( B \) of \( f \) that reflect the indicated signs of the listed coefficients and expressions in the coefficients. Label each of your graphs with \( A \) or \( B \) as appropriate. In case one (or both) cannot exist, explain why not.

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Ans: \( A \): concave down, vertex in first quadrant, y-intercept below x-axis

\( B \): does not exist because \( a, b \) positive forces \(-b/(2a)\) negative

3. (5 points) Each of the graph sketches at right is that of a function of the form \( f : \mathbb{R} \to \mathbb{R} \) where \( f(x) = ax^2 + bx + c \). Complete the table below by entering “+”, “-”, “0”, or “DNE” to indicate the signs (or non-existence) of the coefficients or expressions for the \( f(x) \) corresponding to each of the graph sketches \( A \) and \( B \).

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<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>( B )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
1. If $A = (-1, 4)$ and $B = (6, 2)$ then $P(t) = (-1, 4) + t(7, -2)$ is a parameterization of the line $L$ through $A$ and $B$ such that $P(0) = A$ and $P(1) = B$. What value of the parameter $t$ corresponds to the point on $K$ on $L$ which is between $A$ and $B$ and such that the distance from $A$ to $K$ is 4 times the distance from $B$ to $K$?

**Ans:**

$$|t| = 4|1 - t|$$

$t = 4 - 4t$, $5t = 4$, $t = \frac{4}{5}$ (inside)

$-t = 4 - 4t$, $3t = 4$, $t = \frac{4}{3}$ (outside)

2. Suppose $f(x) = ax^2 + bx + c$. Sketch possible graphs $A$ and $B$ of $f$ that reflect the indicated signs of the listed coefficients and expressions in the coefficients. Label each of your graphs with $A$ or $B$ as appropriate. In case one (or both) cannot exist, give a reason why not.

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$\frac{-b}{2a}$</th>
<th>$b^2 - 4ac$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$-$</td>
<td>$+$</td>
<td>$+$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>$B$</td>
<td>$+$</td>
<td>$0$</td>
<td>$0$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
</tbody>
</table>

**Ans:** $A$: does not exist: concave down and no roots so vertex must be below x-axis so $c$ must be $< 0$ $B$: concave up, symmetric about y-axis, vertex below x-axis.

3. Each of the graph sketches at right is that of a function of the form $f : \mathbb{R} \rightarrow \mathbb{R}$ where $f(x) = ax^2 + bx + c$. Complete the table below by entering “+”, “-”, “0”, or “DNE” to indicate the signs (or non-existence) of the coefficients or expressions for the $f(x)$ corresponding to each of the graph sketches $A$ and $B$.

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$\frac{-b}{2a}$</th>
<th>$b^2 - 4ac$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>$B$</td>
<td>$-$</td>
<td>$+$</td>
<td>$0$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
</tbody>
</table>
1. (5 points) If \( A = (-1, 4) \) and \( B = (6, 2) \) then \( P(t) = (-1, 4) + t(7, -2) \) is a parameterization of the line \( L \) through \( A \) and \( B \) such that \( P(0) = A \) and \( P(1) = B \). What value of the parameter \( t \) corresponds to the point on \( K \) on \( L \) which is between \( A \) and \( B \) and such that the distance from \( A \) to \( K \) is 9 times the distance from \( B \) to \( K \)?

\[
|t| = 9|1 - t|
\]
\[
t = 9 - 9t, \quad 10t = 9 \quad t = \frac{9}{10} \quad \text{(inside)}
\]
\[
-t = 9 - 9t, \quad 8t = 9 \quad t = \frac{9}{8} \quad \text{(outside)}
\]

2. (5 points) Suppose \( f(x) = ax^2 + bx + c \). Sketch possible graphs \( A \) and \( B \) of \( f \) that reflect the indicated signs of the listed coefficients and expressions in the coefficients. Label each of your graphs with \( A \) or \( B \) as appropriate.

In case one (or both) cannot exist, give a reason why not.

\[
\begin{array}{cccc}
| & a & b & c & \frac{-b}{2a} & b^2 - 4ac \\
A & + & 0 & - & + \\
B & - & + & + & - & -
\end{array}
\]

\[
\text{Ans:}
\]

\( A \): concave up, symmetric about y-axis, vertex below x-axis.

\( B \): does not exist: concave down and no roots so vertex must be below x-axis so \( c \) must be \( < 0 \)

3. (5 points) Each of the graph sketches at right is that of a function of the form \( f: \mathbb{R} \rightarrow \mathbb{R} \) where \( f(x) = ax^2 + bx + c \). Complete the table below by entering “+”, “-”, “0”, or “DNE” to indicate the signs (or non-existence) of the coefficients or expressions for the \( f(x) \) corresponding to each of the graph sketches \( A \) and \( B \).

\[
\begin{array}{cccc}
| & a & b & c & \frac{-b}{2a} & b^2 - 4ac \\
A & - & - & - & + \\
B & - & + & 0 & +
\end{array}
\]
1. (5 points) If $A = (-1, 4)$ and $B = (6, 2)$ then $P(t) = (-1, 4) + t(7, -2)$ is a parameterization of the line $L$ through $A$ and $B$ such that $P(0) = A$ and $P(1) = B$. What value of the parameter $t$ corresponds to the point on $K$ on $L$ which is between $A$ and $B$ and such that the distance from $A$ to $K$ is 8 times the distance from $B$ to $K$?

**Ans:**

\[ |t| = 8|1 - t| \]

\[ t = 8 - 8t, \quad 9t = 8, \quad t = \frac{8}{9} \text{ (inside)} \]

\[ -t = 8 - 8t, \quad 7t = 8, \quad t = \frac{8}{7} \text{ (outside)} \]

2. (5 points) Suppose $f(x) = ax^2 + bx + c$. Sketch possible graphs $A$ and $B$ of $f$ that reflect the indicated signs of the listed coefficients and expressions in the coefficients. Label each of your graphs with $A$ or $B$ as appropriate. In case one (or both) cannot exist, give a reason why not.

$$
\begin{array}{cccc}
 a & b & c & \frac{-b}{2a} \quad b^2 - 4ac \\
 A & + & 0 & - 0 & + \\
 B & - & + & + & - & - \\
\end{array}
$$

**Ans:**

$A$: concave up, symmetric about y-axis, vertex below x-axis.

$B$: does not exist: concave down and no roots so vertex must be below x-axis so $c$ must be $< 0$

3. (5 points) Each of the graph sketches at right is that of a function of the form $f : \mathbb{R} \rightarrow \mathbb{R}$ where $f(x) = ax^2 + bx + c$. Complete the table below by entering “+”, “-”, “0”, or “DNE” to indicate the signs (or non-existence) of the coefficients or expressions for the $f(x)$ corresponding to each of the graph sketches $A$ and $B$.

$$
\begin{array}{cccc}
 a & b & c & \frac{-b}{2a} \quad b^2 - 4ac \\
 A & - & - & - & + \\
 B & - & + & 0 & + \\
\end{array}
$$
Quiz 26 (Angle Measure) ANS

MA110 Daily Quiz 26 ANS

Nov. 4, 2015
1) \( X \) degrees = \( \frac{\pi \text{ rad}}{180 \text{ deg}} \) radians. (The answer will be in terms of \( X \))
\[ \text{Ans: } X \text{ deg} \times \frac{\pi \text{ rad}}{180 \text{ deg}} \]

2) \( Y \) radians = \( \frac{180 \text{ deg}}{\pi \text{ rad}} \) degrees. (The answer will be in terms of \( Y \))
\[ \text{Ans: } Y \text{ rad} \times \frac{180 \text{ deg}}{\pi \text{ rad}} \]

3) The vertex of the pie-slice region below is at the center of the circle. The radius of the circle is 9 feet and the length of the circular arc that forms part of the boundary of the shaded region is 7 feet.

\[
\theta \text{ (radians)} = \frac{\text{arc length}}{\text{radius}} = \frac{7}{9} rad.
\]

\[ \text{Ans: } \frac{7}{9} \]
Quiz 27 (Angle Measure)

MA110 Daily Quiz 27

Nov. 6, 2015
Fill in following table with exact answers.

<table>
<thead>
<tr>
<th>θ</th>
<th>cos(θ)</th>
<th>sin(θ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\frac{\pi}{6}$</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\frac{\pi}{4}$</td>
<td>$\frac{\sqrt{2}}{2}$</td>
<td>$\frac{\sqrt{2}}{2}$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>$-1$</td>
<td>0</td>
</tr>
<tr>
<td>$\frac{3\pi}{2}$</td>
<td>0</td>
<td>$-1$</td>
</tr>
</tbody>
</table>
1. Suppose $L$ is the line through $A(-8, 5)$ and $B(7, -3)$.

(a) (3 points) Give a parameterization $P(t) = (x(t), y(t))$ for $L$ such that $P(0) = A$ and $P(1) = B$.

\[ P(t) = (-8, 5) + t((7, -3) - (-8, 5)) = (-8, 5) + t(15, -8) \]
\[ P(t) = (15t - 8, -8t + 5) \]

(b) (3 points) Give a parameterization $Q(t)$ for the line $M$ that is parallel to $L$ and goes through the origin $C(8, -7)$.

\[ Q(t) = (8, -7) + t(15, -8) = (15t + 8, -8t - 7) \]

(c) (3 points) Give a parametrization $N(t)$ for the line $S$ that is perpendicular to $L$ and goes through the point $A$.

\[ N(t) = (-8, 5) + t(15, -8) \perp = (-8, 5) + t(8, 15) \]
\[ N(t) = (-8 + 8t, 5 + 15t) \]

2. (3 points) If the line $S$ has parametric equations $S(t) = (8 + 3t, 2 - 5t)$ and the line $T$ has cartesian equation $y = x + 1$ then the point of intersection of $S$ and $T$ corresponds to the value $t = \frac{7}{8}$. (Show your work.)

\[
\begin{align*}
y &= x + 1 \\
2 - 5t &= (8 + 3t) + 1 \\
t &= \frac{7}{8}
\end{align*}
\]

3. (3 points) Suppose $\mathbb{R}^+$ is the positive numbers and $f : \mathbb{R}^+ \to \mathbb{R}^+$ is given by $f(x) = \frac{1}{3x + 7}$. Show that $f$ is injective.

\[
\begin{align*}
f(a) &= f(b) \\
\frac{1}{3a + 7} &= \frac{1}{3b + 7} \\
3b + 7 &= 3a + 7 \\
3b &= 3a \\
a &= b \text{ therefore if } f(a) = f(b) \text{ then } a = b.
\end{align*}
\]
a) The clock reads 8 : 45. Give the radian measure of the angle through which the minute hand must rotate counterclockwise to overlap the hour hand.

ANS: \( \frac{\pi}{24} \)

b) In the triangle \( ABC \) (as represented in the diagram) the angle \( B \) is .439 radians, the side \( BC \) has length 20 and the side \( AB \) has length 21.

The length of the side \( AC \) (the one opposite the angle \( B \)) is ______.

ANS: \( \sqrt{20^2 + 21^2 - 2 \times 20 \times 21 \times \cos(.439)} \)
1. $P(A)$, the locator point of an angle $A$ is known to be $(0.7660, 0.6428)$. Also, $P(B)$, the locator point of an angle $B$ is known to be $(0.7660, 0.6428)$. Using appropriate trigonometric identities, calculate the locator points for the following angles.

NOTE: You do not need to simplify your answers.

A possibly useful formula is $\cos\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 + \cos(\theta)}{2}}$

(a) The locator point $P(A - B)$ is: (______, ______)  
**Ans:** [1.0, 0.0]

(b) The locator point $P(2A)$ is: (______, ______)  
**Ans:** [0.1739, 0.9848]

(b) The locator point of $\frac{B}{2}$ is: (______, ______)  
**Ans:** [0.9397, 0.3421]
1. If $\triangle ABC$ is a triangle such that $AB = 4$, $BC = 3$, and $AC = 5$ then what is the radian measure of the angle $\angle CAB$?

![Diagram of triangle ABC]

**Ans:** This solution was corrected Dec 14. The original solution had switched the lengths of $AB$ and $BC$

$$3^2 = 5^2 + 4^2 - 2(5)4 \cos(A)$$
$$\cos(A) = \frac{4}{5}$$
$$\arccos\left(\frac{4}{5}\right) = 0.6435\text{rad.}$$
$$A = 0.6435 + 2n\pi \text{ or } A = -0.6435 + 2n\pi$$
$$0 \leq A \leq \pi \text{ so only } A = 0.92729 \text{ is possible.}$$

2. If $1 \leq x \leq 5$ and $\cos(x) = 0.7$ then what are the possible values of $x$?

**Ans:** $\arccos(0.7) = 0.79539 \text{ (rad.) so } x = 0.7 + 2n\pi \text{ or } x = -0.7 + 2n\pi$

$0.7 + 2(-1)\pi = -5.58$, $0.7 + 2(0)\pi = 0.7$ are both less than 1

$0.7 + 2(1)\pi = 6.98$ is greater than 1.

$-0.7 + 2(0)\pi = -0.7 \leq 1$ and $-0.7 + 2(1)\pi = 5.58 > 1$

that is

- $1 \leq 0.7 + 2n\pi \leq 5$ fails for all $n$
- $1 \leq -0.7 + 2n\pi \leq 5$ fails for all $n$

Thus there are no such values of $x$. 

1
At time $t$ hours the position in miles of a car moving at constant speed around a circular race track is

$$P(t) = (1 + 3 \cos(\frac{\pi}{2} - 4\pi t), 2 + 3 \sin(\frac{\pi}{2} - 4\pi t))$$

1. The car moves (clockwise or counterclockwise) around the track.

**Ans:** clockwise

2. The car makes a circuit of the track every ________ minutes.

**Ans:** $4\pi t = 2\pi$, $t = \frac{1}{2}$ hr. (30 min.)

3. In 10 hours the car will have traveled ________ miles (clockwise).

**Ans:** $4\pi \frac{\text{rad}}{\text{hr}} \times 10 \text{ hr} \times \frac{3\text{ mi}}{\text{rad}}$
1. Calculate the tangent line to the algebraic curve

\[ x^2 - 3xy + y^3 + 1 = 0 \]

at the point \( P(2, 1) \)

\[ \text{Ans: } (2)^2 - 3(2)(1) + (1)^3 + 1 = 0 \] so the point is on the curve.

\[ x = u + 2, \ y = v + 1 \]

\[ (u + 2)^2 - 3(u + 2)(v + 1) + (v + 1)^3 + 1 = 0 \]

\[ u - 3v = 0 \] is linear part

\[ (x - 2) - 3(y - 1) = 0 \] is the tangent line.
1. Use two iterations of Newton’s method, starting at $x = 4$ to estimate $\sqrt{17}$. Complete the table below.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$x_n$</th>
<th>$f(x_n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. The coefficient of $x^3y^4$ when $(x - 2y)^7$ is expanded out and simplified is _______. Justify your answer.

**Ans:** 560