MA162: Finite mathematics

Paul Koester

University of Kentucky

September 9, 2013

Schedule:

- Second Web Assign assignment (Chapters 1.3 and 1.4) due on Tuesday, September 10 by 6:00 pm.
- Third Web Assign assignment (Chapter 2.1) due on Friday, September 13 by 6:00 pm.
- “Miniature exam” over Chapter 1 will be given in recitation on either Tuesday, September 10 or Thursday, September 12.

Today we cover chapter 2.1, an introduction to systems of linear equations.
2 equations, 2 unknowns, Unique Solution

Find the solution to the system of equations:

\[
\begin{align*}
x + 2y &= 8 \\
3x + 4y &= 6
\end{align*}
\]

Substitute: Use top equation, \(x = 8 - 2y\)

Another Approach

Replace bottom with bottom - 3\(\times\)top.

\[
\begin{align*}
3x + 4y &= 6 \\
-3x - 3\(2y\) &= -3\times2 \\
0x - 2y &= -18 \Rightarrow y = 9
\end{align*}
\]

Now system is

\[
\begin{align*}
x + 2y &= 8 \\
y &= 9
\end{align*}
\]

Replace top with top - 2\(\times\)bottom.

\[
\begin{align*}
x + 2y &= 8 \\
-2y &= -18 \\
x + 0y &= -10
\end{align*}
\]

Now system is

\[
\begin{align*}
x &= -10 \\
y &= 9
\end{align*}
\]

Solution is \(x = -10, y = 9\).
First Eq.
\[ x + 2y = 8\]
\[ x = 0 \Rightarrow y = \frac{4}{2} \quad y - \text{int}^+ \]
\[ y = 0 \Rightarrow x = 8 \quad x - \text{int}^+ \]

Second
\[ 3x + 4y = 6\]
\[ x = 0 \Rightarrow y = \frac{3}{4} \quad y - \text{int}^+ \]
\[ y = 0 \Rightarrow x = 2 \quad x - \text{int}^+ \]
Find the solution to the system of equations:

\[
\begin{align*}
  x - y &= 4 \\
-3x + 3y &= 5
\end{align*}
\]

**Substitution**
Top Eq \( \Rightarrow \) \( x = y + 4 \)

So bottom
\[
\begin{align*}
  -3(y+4) + 3y &= 5 \\
-3y - 12 + 3y &= 5 \\
-12 &= 5
\end{align*}
\]

**NONSENSE**
System can't have solution.

**Elimination**
Replace bottom Eq with
Bottom + 3 Top
\[
\begin{align*}
  -3x + 3y &= 5 \\
  3x - 3y &= 12 \\
0 + 0y &= 17
\end{align*}
\]
so now bottom Eq is
\( 0 = 17 \)

**NONSENSE!**
Top Eq.
\[ x = y = 4 \]
\[ x = 0 \Rightarrow y = 4 \quad y\text{-int.} \]
\[ y = 0 \Rightarrow x = 4 \quad x\text{-int.} \]

Bottom Eq.
\[ -3x + 3y = 5 \]
\[ x = 0 \Rightarrow y = \frac{5}{3} \quad y\text{-int.} \]
\[ y = 0 \Rightarrow x = \frac{5}{3} \quad x\text{-int.} \]

Lines are parallel, so they will never intersect.
i.e., no solution!
Find the solution to the system of equations:

\[ \begin{align*}
3u + 6v &= 9 \\
2u + 4v &= 6
\end{align*} \]

**Elimination:**

Replace bottom with bottom $- \frac{2}{3}$ top

\[ \begin{align*}
2u + 4v &= 6 \\
-2u - 4v &= -6
\end{align*} \]

\[ 0u + 0v = 0 \]

Bottom equation is $0=0$

So whole system is equivalent to

\[ \begin{align*}
3u + 6v &= 9 \\
0 &= 0
\end{align*} \]

Either way,

Let $v = t$

Then $u + 2t = 9$

\[ u = 9 - 2t \]

Solution is $(9 - 2t, t)$
Each equation is
\[ u + 2v = 3 \]
\[ u = 0 \Rightarrow v = \frac{3}{2}, \quad \text{u-int} \]
\[ v = 0 \Rightarrow u = 3, \quad \text{v-int} \]

The "infinite solutions" or "no solution" cases both arise when the lines have the same slope.

No solution example:
\[ x - y = 4 \]
\[ -3x + 3y = 5 \]

Ratio of \( x \) coeff : \( \frac{-1}{3} \) = Ratio of \( y \) coeff

Infinite solutions
\[ 3u + 6v = 9 \]
\[ 2u + 4v = 6 \]

Ratio of \( u \) coeff : \( \frac{3}{2} \) = Ratio of \( v \) coeff
2 equations, 2 unknowns

Find the value of $k$ for which this system has infinitely many solutions.

\[
\begin{align*}
2x - y &= 2 \\
5x + k \cdot y &= 5
\end{align*}
\]

Then find all solutions corresponding to this value of $k$.

Look at ratios.

\[
\begin{align*}
\frac{x \text{ Coeff}}{x \text{ Coeff}} &= \frac{2}{5} & \text{So} \ x = -\frac{1}{k} \Rightarrow k = \frac{5}{2}
\end{align*}
\]

How do we know "infinite sol" rather than "No sol"?

Ratio of RHS = $\frac{2}{5}$, same as ratios on LHS
Find the value of $k$ for which this system has no solutions.

\[3x - 2y = 3\]
\[6x + k \cdot y = 4\]

Make lines parallel by comparing ratios.

\[\frac{\text{Coeff}_x}{\text{Coeff}_y} = \frac{3}{6} \iff \frac{3}{6} = \frac{-\frac{2}{k}}{1} \Rightarrow k = -2 \cdot \frac{6}{3} = -4\]

No solution or infinite solutions?

Ratio of RHS = \(\frac{3}{4}\) \(\neq\) Ratio of LHS, \(\frac{1}{2}\)

So, No Solution
Communicating Solutions

• The remaining problems in this lesson are story problems.

• For the moment, we only need to "set up" these problems, i.e., we only need to worry about turning the "words" into "math."

• We will "solve" these problems later in chapter 2, as we develop systematic techniques for solving systems of linear equations.

• You must clearly state the names of your variables, units of measure, etc. On exams and recitation quizzes, you will lose points if you do not clearly communicate your solutions!
Michael Perez has a total of $2000 on deposit with two savings institutions. One pays interest at the rate of 6%/year, whereas the other pays interest at the rate of 8%/year. If Michael earned a total of $144 in interest during a single year, how much does he have on deposit in each institution?

Let $x =$ Amount, in dollars, invested @ 6%
$y =$ Amount, in dollars, invested @ 8%

Total Amount invested: $x + y = 2000$

Amount earned @ 6% is $0.06x$
Amount earned @ 8% is $0.08y$

Total Amount earned $0.06x + 0.08y = 144$

\[ \begin{cases} 
  x + y = 2000 \\
  0.06x + 0.08y = 144 
\end{cases} \]
Lawnco produces three grades of commercial fertilizers. A 100-lb bag of grade A fertilizer contains 18 lb of nitrogen, 4 lb of phosphate, and 5 lb of potassium. A 100-lb bag of grade B fertilizer contains 20 lb of nitrogen, 4 lb of phosphate, and 4 lb of potassium. A 100-lb bag of fertilizer C contains 24 lb of nitrogen, 3 lb of phosphate, and 6 lb of potassium. How many 100-lb bags of each of the three grades of fertilizers should Lawnco produce if 26,400 lb of nitrogen, 4900 lb of phosphate, and 6200 lb of potassium are available and all the nutrients are used?

Let \( a \) = number of bags grade A, \( b \) = number of bags grade B, \( c \) = number of bags grade C.

Amount nitrogen: \( 26,400 = 18a + 20b + 24c \)
Amount phosphate: \( 4900 = 4a + 4b + 3c \)
Amount potassium: \( 6200 = 5a + 4b + 6c \)
The management of Hartman Rent-A-Car has allocated $2.25 million to buy a fleet of new automobiles consisting of compact, intermediate-size, and full-size cars. Compacts cost $18,000 each, intermediate-size cars cost $27,000 each, and full-size cars cost $36,000 each. If Hartman purchases twice as many compacts as intermediate-size cars and the total number of cars to be purchased is 100, determine how many cars of each type will be purchased. (Assume that the entire budget will be used.)

Let \( C = \# \text{ compact cars}, \ I = \# \text{ intermediate cars}, \ F = \# \text{ full size cars}. \)

Total Expense: \( 2,250,000 = 18,000 C + 27,000 I + 36,000 F \)

Twice as many compacts as intermediate: \( C = 2I \)

Total # cars: \( 100 = C + I + F \)