MA 111 Section 005 Final Exam Review

Our final exam is Wednesday, May 2, 2012 from 1:00 pm to 3:00 pm in our regular room. The final exam covers topics from Chapter 3 (Fair Division) and Chapter 11 (Symmetry).

1. Five players are dividing a cake among themselves using the lone-divider method. After the divider D cuts the cake into five slices \((s_1, s_2, s_3, s_4, \text{ and } s_5)\), the choosers \(C_1, C_2, C_3, \text{ and } C_4\) submit their bids for these shares.
   
   (a) Suppose the bid sets are: \(C_1\) bids \(\{s_2, s_3\}\), \(C_2\) bids \(\{s_2, s_4\}\), \(C_3\) bids \(\{s_1, s_2\}\), and \(C_4\) bids \(\{s_1, s_3, s_4\}\). Describe THREE different fair divisions of the cake.
   
   (b) Suppose the bid sets are: \(C_1\) bids \(\{s_1, s_4\}\), \(C_2\) bids \(\{s_2, s_4\}\), \(C_3\) bids \(\{s_2, s_4, s_5\}\), and \(C_4\) bids \(\{s_2\}\). Find a fair division of the cake.
   
   (c) Explain why there is ONLY ONE fair division of the cake in part (b).

2. Andrea, Barry, and Cory are dividing an estate which consists of a house, a car, and a boat. They will divide the goods using the Method of Sealed Bids. Here are their bids:

<table>
<thead>
<tr>
<th>Item</th>
<th>Andrea</th>
<th>Barry</th>
<th>Cory</th>
</tr>
</thead>
<tbody>
<tr>
<td>House</td>
<td>$230,000</td>
<td>$220,000</td>
<td>$195,000</td>
</tr>
<tr>
<td>Car</td>
<td>$9,000</td>
<td>$12,000</td>
<td>$14,000</td>
</tr>
<tr>
<td>Boat</td>
<td>$30,000</td>
<td>$40,000</td>
<td>$35,000</td>
</tr>
</tbody>
</table>

   (a) Determine the fair-dollar share for each of Andrea, Barry, and Cory.
   
   (b) Which items (if any) does Andrea get? How much money does Andrea have to pay in or receive from the estate?
   
   (c) Which items (if any) does Barry get? How much money does Barry have to pay in or receive from the estate?
   
   (d) Which items (if any) does Cory get? How much money does Cory have to pay in or receive from the estate?
   
   (e) Is there a surplus? If so, how much?
   
   (f) Describe the final allocation of the goods. Include which goods each person received as well as the net amount of cash the person received. Be sure to check that each person received at least 33.33% of the total.

3. Three college roommates, Mark, Jim, and Vinny, are graduating and are getting ready to move out. While living together, they purchased 18 CDs. 9 of the CDs are jazz CDs, 6 are country, and the remaining 3 are western CDs. Mark only likes jazz. Jim likes country and western, but doesn’t like jazz. Vinny likes country and jazz, but does not like western. The roommates decide to use the method of markers to divide up the CDs. The CDs are lined up in random order, as shown.

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   (a) Determine where each of Mark, Jim, and Vinny, will place their markers.
   
   (b) Now determine how many of each type of CD each of the three roommates will get.
   
   (c) Which CDs are left over?
4. Gail and Frank are going to split a $24 pizza. They each paid $12 for it. One half of the pizza is pepperoni. The other half is mushrooms. Gail likes pepperoni twice as much as she likes mushrooms. Frank likes mushrooms twice as much as he likes pepperoni. Frank and Gail do not know each others likes/dislikes. They decide to divide the pizza using the Divider-Chooser method.

(a) Suppose that Gail is the divider. How will she divide the pizza? Which slice will Frank choose? How much does Frank think his slice is worth?

(b) Now suppose that Frank is the divider. How will he divide the pizza? Which slice will Gail choose? How much does Gail think her slice is worth?

(c) If Frank has a choice between being the divider or being the chooser, would Frank be better off being the divider or the chooser?

(d) Suppose that Frank discovers Gail’s likes and dislikes, but Gail still does not know Franks’s likes or dislikes. Would it be in Frank’s best interest to be the divider or the chooser? (Do the division with Frank as the divider, then with Frank as the chooser, and compare. These divisions may be different from those in parts (a) and (b) since Frank knows Gail’s likes and dislikes).
5. For each of these finite figures, identify the symmetry type. (Type $Z_n$ or $D_n$ for appropriate $n$)

(a) A figure with $Z_3$ symmetry.
(b) A figure with $D_4$ symmetry.
(c) A figure with $D_\infty$ symmetry.
(d) A figure with $Z_7$ symmetry.

6. Draw finite figures with each of the requested symmetry types. (On the exam, you could be asked to draw a figure “free-style”, or you could be given the basic motif.)

Symmetry types for Border Patterns

<table>
<thead>
<tr>
<th></th>
<th>Trans</th>
<th>H. Reflect</th>
<th>V. Reflect</th>
<th>Half Turn</th>
<th>Glide</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>11</td>
</tr>
<tr>
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<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
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<td>Y</td>
<td>N</td>
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<td>Y</td>
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<tr>
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<td>Y</td>
<td>Y</td>
<td>Y</td>
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</tr>
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<td>N</td>
<td>N</td>
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<td>7</td>
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<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>mg</td>
</tr>
</tbody>
</table>
7. For each of these “dance step” patterns, identify the symmetry type.
   (a) Type:
   
   ![Image](footprints_a.png)

   (b) Type:
   
   ![Image](footprints_b.png)

   (c) Type:
   
   ![Image](footprints_c.png)

8. In the grid, draw a border pattern with the requested symmetry type. (You may choose the length of the translation vector, the location of any vertical axes of symmetry, the location of any rotocenters, and the basic motif. On the exam, you could be asked to do a border pattern “free-style”, or you may be given the motif (like on the in-class worksheets))

   ![Grid with symmetry type 1g](grid_1g.png)

   ![Grid with symmetry type m1](grid_m1.png)

   ![Grid with symmetry type mg](grid_mg.png)

   ![Grid with symmetry type mg](grid_mg.png)