Symmetries of Border Patterns

Now let’s look at symmetries of border patterns — these are patterns in which a basic motif repeats itself indefinitely (forever) in a single direction (say, horizontally), as in an architectural frieze, a ribbon, or the border design of a ceramic pot.
Symmetries of Border Patterns

What symmetries does this pattern have?
Symmetries of Border Patterns

You can slide, or translate, this pattern by the basic translation shown above.
Symmetries of Border Patterns

You can slide, or translate, this pattern by the basic translation shown above. This translation is the smallest translation possible; all others are multiples of this one, to the right and to the left.
Symmetries of Border Patterns

You can slide, or **translate**, this pattern by the basic translation shown above. This translation is the smallest translation possible; all others are multiples of this one, to the right and to the left. So this border pattern only has **translational symmetry**.
Symmetries of Border Patterns

What symmetries does this pattern have?
Symmetries of Border Patterns

You can translate this pattern by the basic translation shown above.
Symmetries of Border Patterns

You can translate this pattern by the basic translation shown above. This translation is the smallest translation possible; all others are multiples of this one, forward and backward.
Symmetries of Border Patterns

You can also match the pattern up with itself by a combination of a reflection followed by a translation parallel to the reflection. This is called a **glide reflection**.
Symmetries of Border Patterns

You can also match the pattern up with itself by a combination of a reflection followed by a translation parallel to the reflection. This is called a glide reflection. So this border pattern has both translational symmetry and glide reflectional symmetry.
Symmetries of Border Patterns

What symmetries does this pattern have?
You can slide, or translate, this pattern by the basic translation shown above.
Symmetries of Border Patterns

You can slide, or translate, this pattern by the basic translation shown above. This translation is the smallest translation possible; all others are multiples of this one, to the right and to the left.
There are infinitely many centers of 180 degree rotational symmetry. Here is one type of location of a rotocenter.
Symmetries of Border Patterns

And here is another type of location of a rotocenter.
Symmetries of Border Patterns

And here is another type of location of a rotocenter. But this pattern has no reflectional symmetry or glide reflectional symmetry.
Symmetries of Border Patterns

And here is another type of location of a rotocenter. But this pattern has no reflectional symmetry or glide reflectional symmetry. So this border pattern only has translational symmetry and 2-fold (or half-turn) rotational symmetry.
Symmetries of Border Patterns

What symmetries does this pattern have?
Symmetries of Border Patterns

You can translate this pattern by the basic translation shown above.
Symmetries of Border Patterns

You can translate this pattern by the basic translation shown above. This translation is the smallest translation possible; all others are multiples of this one, to the right and to the left.
Symmetries of Border Patterns

This pattern has one horizontal axis of reflectional symmetry
Symmetries of Border Patterns

This pattern has one horizontal axis of reflectional symmetry but infinitely many vertical axes of reflectional symmetry, that have two types of locations.
Symmetries of Border Patterns

Because there are translations and horizontal reflections, we can combine them to get glide reflections. Here is one type of glide reflection. Others use the same reflection axis but multiples of this translation, to the right and to the left.
Symmetries of Border Patterns

There are infinitely many centers of 180 degree rotational symmetry. Here the two types of locations of rotocenters.
Symmetries of Border Patterns

So this border pattern has **translational, horizontal reflectional, vertical reflectional, and 2-fold rotational** symmetry. Even though glide reflections also work, our text states that we don’t say this pattern has “glide reflectional symmetry” because the glide reflections are in this case just a consequence of translational and the horizontal reflectional symmetry.