This is a closed book exam. No books or notes are to be used during the exam. You may
use a graphing calculator if it does not have symbolic manipulation capabilities. However, any
device capable of electronic communication (cell phone, pager, etc.) must be turned off and
out of sight during the exam.

Each question is followed by space to write your answer. Please write your solutions neatly
in the space below the question. Partial credit for a problem will be given only when there
is coherent written evidence that you have solved part of the problem. In particular, answers
that are obtained simply as the output of calculator routines will receive no credit. Show your
work. Answers without justification will receive no credit.

Name: ________________________________

Section: ________

Last four digits of student identification number: ________

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1) (23 pts) Compute the following integrals:

(a) (7 pts) \[ \int x \sin(5x) \, dx. \]

(b) (7 pts) \[ \int x \sqrt{4 - x^2} \, dx. \]

(c) (9 pts) \[ \int_{0}^{1} \sqrt{4 - x^2} \, dx. \]
2) (16 pts) Let $R$ be the name of the region enclosed by the curves $y = 3x^2$ and $y = 12$.
(a) (3 pts) Sketch and shade in the region $R$.

(b) (6 pts) Compute the area of the region $R$.

(c) (7 pts) Set up an integral that computes the volume of the solid that is obtained by rotating the region $R$ about the $x$-axis. State whether you are using the method of washers, the method of cylindrical shells or another method.
3) (4 pts) Obtain the exact value of \( \int_{4}^{6} \left( \frac{1}{x} + \frac{1}{3-x} \right) \, dx \) in terms of logarithms. Simplify your answer as much as possible.

4) (11 pts) Let \( C \) be the curve defined parametrically by \( x = e^{2t}, \ y = e^{3t}, \ 0 \leq t \leq 1 \).
   (a) (4 pts) Eliminate the parameter \( t \) to find the Cartesian equation of \( C \).

   (b) (7 pts) Find the length of \( C \).
5) (a) (6 pts) Find the sum or show the divergence of \( \sum_{n=0}^{\infty} (-1)^n \frac{3^n}{5^{n+1}} \).

(b) (6 pts) Determine whether the series \( \sum_{k=1}^{\infty} (-1)^k \frac{\sin(k)}{k^3 + 1} \) is convergent. Give the name of any of the tests you apply and show how you apply them.

(Problem 5 continued on next page)
(c) (8 pts) Determine the interval of convergence of the power series \( \sum_{n=0}^{\infty} \frac{n(x + 3)^n}{2^n} \). Remember to consider the endpoints.
6) (8 pts) Find the Maclaurin series of \( f(x) = \frac{1}{(x + 5)^2} \) and determine its radius of convergence. Express your answer using sigma notation.

7) (8 pts) Solve the initial-value problem \( y' = 2x(y^2 + 1), \ y(0) = 1 \). Express your answer explicitly as a function of \( x \).
8) (9 pts) This problem considers the differential equation \( \frac{dP}{dt} = P(P - 2) \).

   (a) (2 pts) Find all the equilibrium solutions.

   (b) (5 pts) Sketch the portion of the direction field on the lines \( P = 0, P = 1, P = 2 \) and \( P = 3 \).

   (c) (2 points) Plot the solution curve with \( P(0) = .7 \) on the graph for part (b).

9) (10 pts) Let \( C \) be the polar curve \( r = \theta / \pi, \ 0 \leq \theta \leq 2\pi \).

   (a) (5 pts) Sketch the graph of \( C \). Include on your graph the polar coordinates of the intercepts of \( C \) with the \( x \)- and \( y \)-axes.

   (Problem 9 continued on next page)