Find a pair of numbers whose sum and product are both equal to 6.

(Use symbolic notation and fractions where needed. Give your answer in the form of a comma separated list.)

The numbers are

\[ x, y \]  
\[ \text{Sum is } 6 \]
\[ x + y = 6 \]
\[ \text{Product is } 6, \quad x \cdot y = 6 \]

\[ y = 6 - x \]
\[ x \cdot y = 6 \]
\[ x (6 - x) = 6 \]

\[ 6x - x^2 = 6 \]
\[ -6x + x^2 \]
\[ x^2 - 6x + 9 + 6 = 0 + 9 \]
\[ (x - 3)^2 + 6 - 6 = 9 - 6 \]
\[ (x - 3)^2 = 3. \]
\[ x - 3 = \pm \sqrt{3} \]
\[ x = 3 \pm \sqrt{3}. \]

Try \( x = 3 + \sqrt{3}. \)
\[ y = 6 - x = 6 - (3 + \sqrt{3}) \]
\[ = 3 - \sqrt{3} \]

\( x = 3 + \sqrt{3}, \quad y = 3 - \sqrt{3}. \)

Check. \( x + y = 6. \) \( x \cdot y = \? \)
Find the inverse $f^{-1}$ of $f(x) = \frac{x - 4}{1 + 5x}$

(Use symbolic notation and fractions where needed.)

$$f^{-1}(x) =$$

**Solution:**

$$y = \frac{x - 4}{1 + 5x}$$

$$y (1 + 5x) = x - 4$$

$$5yx + y - x = x - 4 - x - y$$

$$5yx - x = -4 - y$$

$$x (5y - 1) = -4 - y$$
\[ x = \frac{-4-y}{5y-1} = \frac{4+y}{1-5y} \]

\[ f^{-1}(x) = \frac{4+x}{1-5x} \]

Check. - Graph.

\[ f(0) = -4. \text{ Try } f^{-1}(-4) = 0 \]
Let \( f(x) = x^5 + x + 8 \). Find the value of the inverse function at a point.

(Use symbolic notation and fractions where needed.)

\[
\begin{align*}
  f^{-1}(254) & = \\
  \text{help (fractions)} & 
\end{align*}
\]

Try to solve \( x^5 + x + 8 = 254 \).

 נכונה: Solving this equation is hard.

Hope today you are lucky.

\[
\begin{align*}
  f(0) & = 8 \times f(1) = 10 \times \\
  f(2) & = 42 \times f(3) = 243 + 3 + 8 \\
  & = 254 \smile \\
  f^{-1}(254) & = 3.. 
\end{align*}
\]
A bug is located at the point (5, 0) at time $t = 0$ and crawls at the rate of 6 units/minute in the counterclockwise direction along the circle centered at the origin of radius 5.

Find the coordinates $(x, y)$ which give the location of the bug after 26 minutes.

$x =$  
$y =$

After how many minutes will the bug first return to the location $(5, 0)$?

minutes

Give the coordinates $(x(t), y(t))$ at an arbitrary time $t \geq 0$.

$x(t) =$  
$y(t) =$

**Solution:**

After 26 minutes, the bug has moved $26.6$ units. Location is $(5 \cos(\frac{156}{5}), 5 \sin(\frac{156}{5}))$. After $t$ minutes, $\sqrt{5^2 \cos^2(\frac{156}{5}) + 5^2 \sin^2(\frac{156}{5})}$.
\((5 \cos (\frac{6\pi}{5}), 5 \sin (\frac{6\pi}{5}))\).

1st return after \(\frac{2\pi}{5}\) minutes.
(1 point) local/rmb-problems/exp-alg.png

A function $f$ is given by the formula $f(x) = A \cdot e^{kx}$ for constants $A$ and $k$. We also know that $f(0) = 11$ and $f(4) = 8$.

Find numerical values for the constants $A$ and $k$.

\[
A = \quad , \quad k = \]

```
help (numbers).
```

The function $f$ is ?

\[
\text{Solution:}
\]

\[
f(0) = 11 = A \cdot e^{0} = A
\]

\[
f(4) = 8 = A e^{4k}
\]

\[
A = 11. \quad 8 = 11 e^{4k}
\]

\[
\ln \left( \frac{8}{11} \right) = \ln \left( e^{4k} \right) = 4k \ln(e).
\]

\[
\ln \left( \frac{8}{11} \right) = 4k.
\]
\[ k = \frac{1}{4} \ln \left( \frac{8}{11} \right). \]
Let $F$ be the function whose graph is shown below. Evaluate each of the following expressions.

*(If a limit does not exist or is undefined, enter "DNE".)*

1. $\lim_{x \to -1^-} F(x) = \text{[ ]}$
2. $\lim_{x \to -1^+} F(x) = \text{[ ]}$
3. $\lim_{x \to -1} F(x) = \text{[ ]}$
4. $F(-1) = \text{[ ]}$
5. $\lim_{x \to 1^-} F(x) = \text{[ ]}$
6. $\lim_{x \to 1^+} F(x) = \text{[ ]}$
7. $\lim_{x \to 1} F(x) = \text{DNE}$
8. $\lim_{x \to 3} F(x) = \text{[ ]}$
9. $F(3) = \text{DNE}$

The graph of $y = F(x)$. 

![Graph of $y = F(x)$](image-url)
Evaluate the limit assuming that \(\lim_{x \to 2} g(x) = 10\):

\[
\lim_{x \to 2} \frac{g(x)}{x^2} = 10.
\]

**Solution:**

\[
\lim_{x \to 2} \frac{g(x)}{x^2} = \lim_{x \to 2} g(x) \cdot \lim_{x \to 2} \frac{1}{x^2} = 10 \cdot 4 = 40.
\]

Provided \(\lim_{x \to 2} x^2 = 4\) is not 0. But

\[
\lim_{x \to 2} x^2 = 4,
\]

since \(g(x) = x^2\) is continuous.
Thus
\[ \lim_{x \to 2} \frac{f(x)}{g(x)} = \lim_{x \to 2} \frac{x^2}{x^2} = \frac{10}{4} = \frac{5}{2}. \]